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The Establishment of the Absolute Scale of Temperature below 1° K.

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ABSTRACT. Experiments are discussed establishing the relation of the absolute thermodynamic scale of temperature below 1° K. to the magnetic scale obtained by the extrapolation of Curie's law. The experimental results on iron ammonium alum, manganous ammonium sulphate, and chrome potassium alum are compared with the theoretical calculations of Onsager and Van Vleck.

§ 1. INTRODUCTION

THE range of temperatures below 1° K. forms a distinct region of low temperature technique, in which the temperatures are attained and measured magnetically, in contrast with the methods dependent on the manipulation of gases used at higher temperatures. Temperature measurements below 1° K. are made by determining the magnetic moment of a specimen of paramagnetic salt in a small magnetic field. At high temperatures this is proportional to the magnetic field and inversely proportional to the absolute temperature (Curie's law); the scale of temperature obtained by the extrapolation of this law to the low temperatures reached by the adiabatic demagnetization method is termed the magnetic or Curie scale. Because of demagnetizing effects, the magnetic moment of a given mass of salt at a fixed field and temperature depends on the shape of the specimen. To avoid a multiplicity of magnetic temperature scales, it is the practice to correct the magnetic moment of a specimen to that which would be displayed by a spherical sample (Kurti and Simon 1938, Hull 1947). It is the scale of temperature derived in this way that we shall refer to as the magnetic temperature. It will be denoted by T^*. Up to the present most of the results obtained in this region of temperature have been expressed in terms of this scale, although it is known that at sufficiently low temperatures departures from Curie's law render it more or less invalid. An extreme case will illustrate this. Adiabatic demagnetization of iron ammonium alum from a field of 15,000 oersted at 1° K. will bring the specimen to a temperature at which its susceptibility is a maximum. Demagnetization from a greater field at 1° K. will bring the specimen to a lower absolute temperature, but to a higher temperature on the magnetic scale, an absurd state of affairs.

In the experiments considered here the relation of the magnetic scale to the absolute scale of temperature has been determined for three different paramagnetic salts. Besides enabling temperature measurements made with these salts to be

corrected to the absolute scale, a comparison of the experimental results with theory enables us to calculate the separation of the energy levels of the magnetic ions in these salts, and to predict to what extent theoretical corrections can safely be applied to magnetic temperature measurements made with other salts. The method used, which was the same in all three cases, was suggested by Keesom (1934) and by Kurti and Simon (1935 a), who first applied it to iron ammonium alum (Kurti and Simon 1935 b). The measurements on manganous ammonium sulphate were made by Dr. R. A. Hull and the writer, and those on potassium chrome alum by Dr. B. Bleaney.

§2. EXPERIMENTAL METHOD

The method consists in determining the heat input to the salt and the change in its entropy as functions of the magnetic temperature. Consider a reversible change made at a temperature T^* on the magnetic scale, involving a small heat input dQ and a small entropy change dS. Then the absolute temperature T corresponding to the magnetic temperature T^* is given by $T=dQ/dS$. Thus if graphs of Q and S as functions of the magnetic temperature T^* are prepared, the ratio of their slopes at a given value of T^* gives the corresponding absolute temperature T. Other related quantities, such as the true specific heat, dQ/dT, can then be evaluated.

The specimens of paramagnetic salt used in the experiments were prepared from "Analar" material. As it is most important that there should be no loss of water of crystallization, the specimens were prepared by selecting large crystals of the salt, grinding them to powder and immediately compressing the powder in a hydraulic press to a pressure of about 2 tons/cm^2. This process produced cylinders which could be turned in a lathe and finally ground to a spheroidal shape, usually 5 cm. long and 1·25 cm. diameter. As the density of the specimens was within 1% to 2% of the crystalline density, all the calculations of demagnetizing effects used below have been based on the crystalline density.

The entropy curve is determined in a series of experiments in which a specimen of the salt to be examined is magnetized at a known field and temperature, producing a calculable change of entropy, followed by adiabatic demagnetization. The measurement of the resulting temperature on the magnetic scale is made by comparing the magnetic moment of the specimen in a field of a few oersted with its moment in the same field at the known initial temperature. The graph of entropy against magnetic temperature is then obtained by plotting this temperature against the calculated change of entropy occurring during the isothermal magnetization. In principle, this change of entropy could be found experimentally, instead of by calculation, by measuring the heat evolved during magnetization (for example, by measuring the increased evaporation of helium from the cryostat), but in practice it is difficult to secure reversible heat exchange between the specimen and the cryostat, and the entropy change is therefore calculated. At 1° K. the salts used can be considered as ideal paramagnetics in which interaction effects can be ignored.† Tables of entropy of ideal paramagnetics are available (Hull and Hull 1941). The results of two series of such experiments, made on iron ammonium alum and on manganous ammonium sulphate, are shown in Figures 1 and 2. The temperature of magnetization was usually between 0·9° K. and 1·0° K.

† A correction for interaction effects was in fact applied in the calculations on iron ammonium alum.

and the magnetic fields used ranged between 740 oersted and 40,000 oersted. Values of the (magnetic field)/temperature ratio corresponding to different values of entropy are shown at the right of Figure 2.

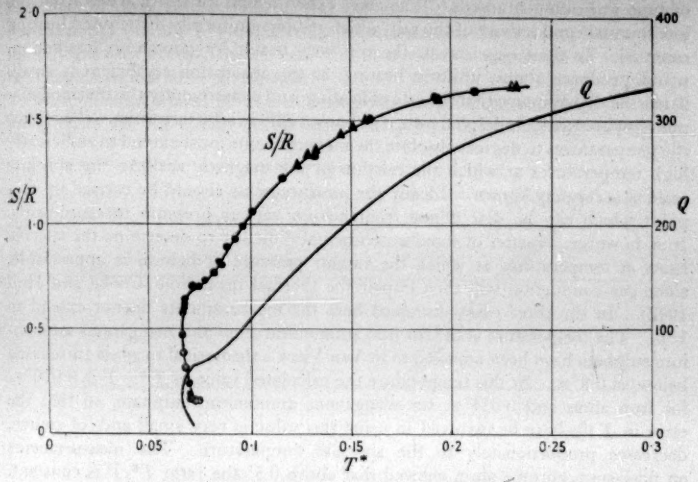

Figure 1. Entropy and heat content of iron ammonium alum, plotted against magnetic temperature. Units of heat content are minutes of γ-ray heating.

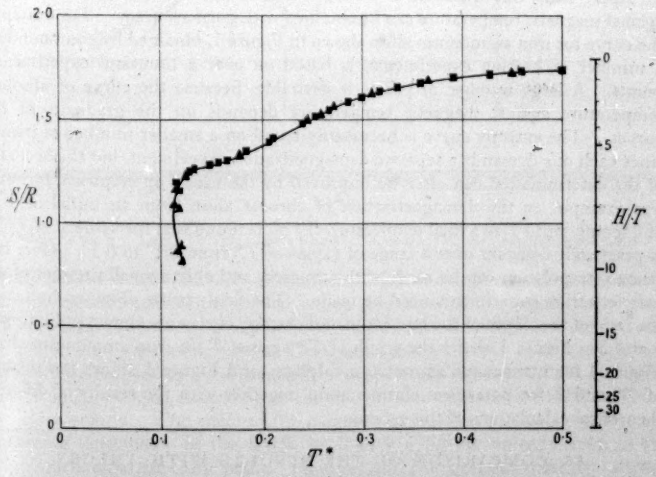

Figure 2. Entropy of manganous ammonium sulphate plotted against magnetic temperature. Right-hand column shows corresponding values of H/T, in kilo-oersted/degree.

The curve of heat input against magnetic temperature is obtained by demagnetizing the salt to as low a temperature as possible, followed by heating at a uniform rate, with observation of the magnetic temperatures at fixed intervals of time as heating proceeds. The chief experimental difficulty arises from the low thermal conductivity of the salt, which renders uniformly distributed heating essential. In these experiments the salts were heated by gamma ray irradiation, which produces almost uniform heating, as the absorption coefficient is small. It has the disadvantage that the rate of heating, and consequently the thermodynamic temperature scale derived from it, is known only in arbitrary units. To reduce the temperatures to degrees absolute the measurements must extend to sufficiently high temperatures at which the relation of the magnetic scale to the absolute scale is accurately known. Ideally the measurements should be carried up to a point which can be determined from helium vapour pressure measurements. It is, however, a matter of some experimental difficulty to determine the specific heats at temperatures at which the vapour pressure of helium is appreciable, since gas conduction will then impair the thermal insulation (Cooke and Hull 1942). In the three cases discussed here the measurements do not extend to 1° K. The temperature scales for iron ammonium alum and manganous ammonium sulphate have been adjusted to fit Van Vleck's theoretical formula (discussed below) at 0·5° K. At this temperature the calculated value of $T^* - T$ is 0·010° K. for iron alum and 0·024° K. for manganous ammonium sulphate, so that the error in T likely to be involved in using this value is very small and, of course, decreases proportionately to the absolute temperature. The measurements on potassium chrome alum showed that above 0·5° the ratio T^*/T is constant. It was therefore assumed that at these temperatures the two scales coincide. For this salt the Van Vleck formula gives $T^* - T = 0{\cdot}009°$ K. at 0·5° K.

Apart from this question of absolute magnitude, the curve of heat input against magnetic temperature can be obtained with great accuracy. For example, the curve for iron ammonium alum shown in Figure 1, obtained by compounding a number of heating experiments, is based on over a thousand experimental points. A large number of points is desirable, because the curve of absolute temperature against magnetic temperature depends on the gradients of the curves. The entropy curve is necessarily based on a smaller number of points, since each one demands a separate demagnetization experiment, but the accuracy of the determination can often be improved by the use of an empirical formula. For example, in the demagnetization of chrome alum from an initial field H_i and temperature T_i to a final temperature T_f^*, it is found that the value of $H_i T_f^*/T_i$ is practically constant over a range of values of T_f^* from 0·1° to 0·3°. Over this range interpolation can be made with accuracy, and only a small number of demagnetization experiments need be made. The final results obtained by taking the ratio of the slopes of the heat input and entropy curves are shown in Figures 3, 4 and 5. Figure 3 shows the graph of T^* against T for iron ammonium alum, Figure 4 for manganous ammonium sulphate, and Figure 5 shows the relation of T^* and T for potassium chrome alum, together with the results of different theoretical calculations of this relation.

§3. COMPARISON OF THE RESULTS WITH THEORY

Curie's law will be obeyed by a paramagnetic salt so long as it can be regarded as an assembly of free magnetic dipoles. Two effects will cause departures from

Curie's law, the action on the dipoles of the electric field of the crystalline lattice and the magnetic interaction of the dipoles. Owing to the high magnetic dilution of the salts considered, exchange forces will be neglected. We have to consider the effect of these interactions on the entropy of the salts and on the magnetic susceptibility (i.e. on T^*) and also the occurrence of a Curie point, below which ferromagnetism occurs, found near the susceptibility maximum of iron alum or manganous ammonium sulphate.

In iron ammonium alum and manganous ammonium sulphate, for which $J=\frac{5}{2}$, there are six energy levels of the magnetic dipoles, giving an entropy

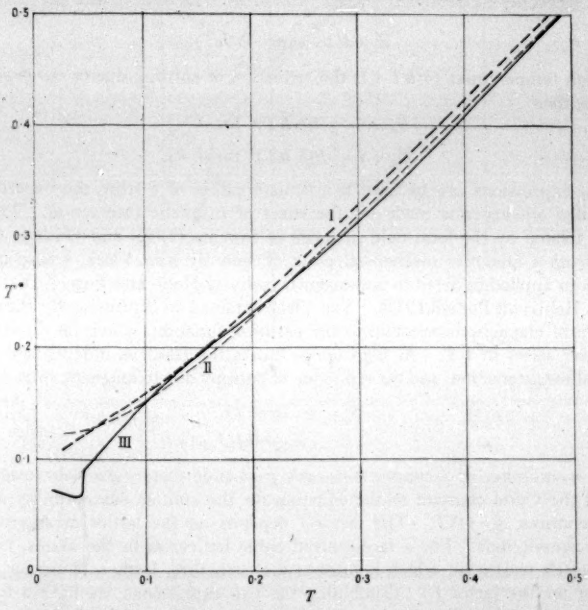

Figure 3. Magnetic temperature against absolute temperature for iron ammonium alum. Curve I, Onsager's theory; Curve II, Van Vleck's theory; Curve III (full line), experimental results.

in zero field of $R \ln 6$, $(1{\cdot}792R)$, per mole. If as a first approximation we assume that the electric field of the crystal lattice possesses cubic symmetry, its effect is to split the levels into two groups, one of two-fold and one of four-fold degeneracy, with an energy separation δ (Van Vleck and Penney 1934). The entropy will therefore be reduced at a sufficiently low temperature either to $R \ln 4$, $(1{\cdot}386R)$, or to $R \ln 2$, $(0{\cdot}693R)$, according as the four-fold or the two-fold level possesses the lower energy. The effect of the magnetic interaction will be to remove the remaining degeneracy of the levels, producing a further entropy drop in the neighbourhood of the Curie point. Figures 1 and 2 suggest that these two entropy reductions can be considered to occur in roughly separate stages, and that in iron alum the crystalline field reduces the entropy to $R \ln 2$, i.e. the four-fold leve-

is uppermost, while in manganous ammonium sulphate the two-fold level is uppermost. Thus, assuming the action of the crystalline field to be independent of the magnetic interaction, its effect is represented by the partition function

$$Z_c = 2\{2 + \exp(-\delta/\boldsymbol{k}T)\},$$

δ being positive for iron alum and negative for manganous ammonium sulphate.

An electric field of cubic symmetry will not affect the energy levels of chrome alum, for which $J = \frac{3}{2}$ (Penney and Schlapp 1932), but departures from cubic symmetry will split the levels into two groups of two-fold degeneracy (Schlapp and Penney 1932, Hebb and Purcell 1937). The corresponding partition function is

$$Z_c = 2\{1 + \exp(-\delta/\boldsymbol{k}T)\}.$$

At high temperatures ($\delta/\boldsymbol{k}T \ll 1$) the reduction of entropy due to the crystalline field is then

$$S_0 - S = \tfrac{1}{9} R(\delta/\boldsymbol{k}T)^2 \text{ for } J = \tfrac{5}{2}$$

and

$$S_0 - S = \tfrac{1}{8} R(\delta/\boldsymbol{k}T)^2 \text{ for } J = \tfrac{3}{2}.$$

These expressions can be used to calculate values of δ from the experiments, provided allowance is made for the effect of magnetic interaction. This has been treated by the local field methods of Lorentz (1916) and Onsager (1936), and from a quantum mechanical point of view by Van Vleck, whose method has been applied in detail to paramagnetic salts by Hebb and Purcell (Van Vleck 1937, Hebb and Purcell 1937). Van Vleck obtained an expression for the contribution of magnetic interaction to the partition function, which he expanded in a power series in $1/T$. At high temperatures the result is independent of the crystalline interaction, and the reduction of entropy due to magnetic interaction is

$$S_0 - S = Rf\tau^2/T^2,$$

where

$$\tau = ng^2\beta^2 J(J+1)/R.$$

Here n = number of magnetic ions/cm^3, g = Landé factor, β = Bohr magneton. $\tau/3$ is the Curie constant in the equation for the volume susceptibility at high temperatures, $\chi = \tau/3T$. The factor f depends on the lattice arrangement of the magnetic ions. For a face-centred cubic lattice, as in the alums, $f = 1{\cdot}20$. (Onsager's treatment, which ignores crystal structure, leads to a similar result, except for the factor f.) Combining the two expressions, we have a formula for the reduction of entropy at high temperatures in which all the quantities save δ are known. Comparison with the experimental results gives the following values of δ:

Iron ammonium alum	$\tau = 0{\cdot}0472°$,	$\delta/\boldsymbol{k} = \ \ 0{\cdot}20°$.
Manganous ammonium sulphate	$\tau = 0{\cdot}062°$,	$\delta/\boldsymbol{k} = -0{\cdot}33°$.
Potassium chrome alum	$\tau = 0{\cdot}0204°$,	$\delta/\boldsymbol{k} = \ \ 0{\cdot}24°$.

§ 4. THE MAGNETIC SUSCEPTIBILITY

Unlike the entropy, the magnetic susceptibility depends on the shape of the specimen. At high temperatures the different methods of calculating magnetic interaction all reduce to the Lorentz local field formula (Lorentz 1916):

$$\frac{T^*}{T} = 1 - \frac{1}{3}\left(\frac{4\pi}{3} - D\right)\frac{\tau}{T},$$

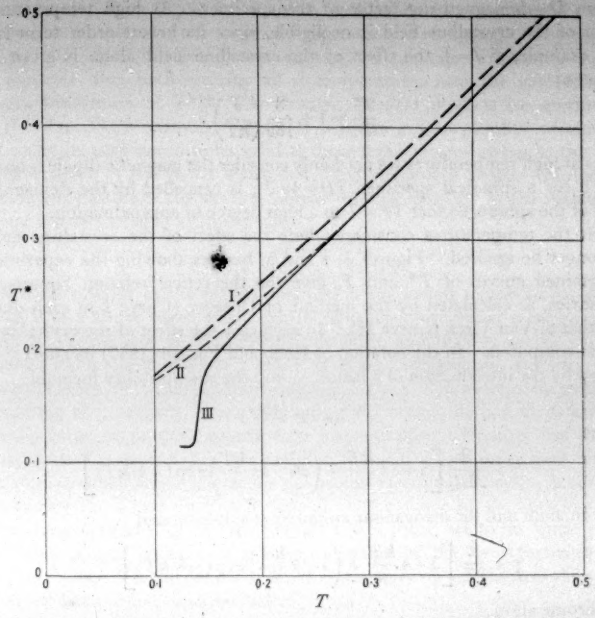

Figure 4. Magnetic temperature against absolute temperature for manganous ammonium sulphate. Curve I, Onsager's theory; Curve II, Van Vleck's theory; Curve III (full line), experimental results.

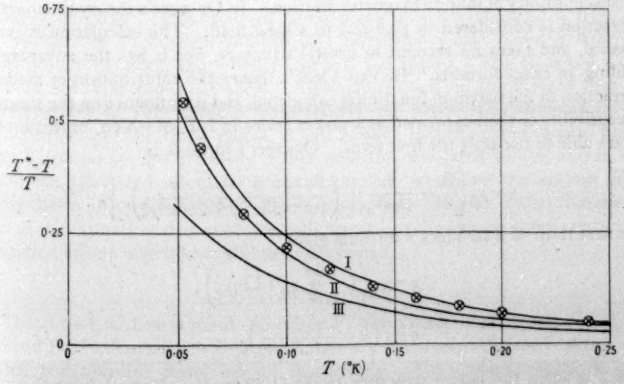

Figure 5. Relation between magnetic temperature and absolute temperature for potassium chrome alum. Curve I, Onsager's theory; Curve II, Van Vleck's theory; Curve III. Lorentz' theory. Crosses are experimental results.

where D = demagnetizing factor of the specimen. At high temperatures the effect of the crystalline field is negligible, since its lowest order term is $1/T^2$. For example, if $J=\frac{5}{2}$, the effect of the crystalline field alone is given by the equation

$$\frac{T^*}{T}=1+\frac{8}{189}\left(\frac{\delta}{\boldsymbol{k}T}\right)^2.$$

Thus at high temperatures we need only consider the magnetic dipole interaction, which for a spherical specimen, $(D=4\pi/3)$, is cancelled by the demagnetizing field of the sphere, so that $T^*=T$ to a high degree of approximation.

At the temperatures considered here the effect of the crystalline field can no longer be ignored. Figures 3, 4 and 5, besides showing the experimentally determined curves of T^* and T, give the theoretical relation between these quantities, as calculated by the method of Onsager (Curve I in each diagram) and that of Van Vleck (Curve II). In each case the effect of the crystalline field is first computed. In the notation of Hebb and Purcell (1937) its effect is represented by the introduction of a factor, γ, into the susceptibility formula:

$$\chi_0=\gamma\tau/3T,$$

where

$$\gamma=\frac{2}{21Z_c}\left[5+32\frac{\boldsymbol{k}T}{\delta}+\left(26-32\frac{\boldsymbol{k}T}{\delta}\right)\exp\left(-\delta/\boldsymbol{k}T\right)\right]$$

for iron alum and for manganous ammonium sulphate, and

$$\gamma=\frac{2}{5Z_c}\left[3+4\frac{\boldsymbol{k}T}{\delta}+\left(3-4\frac{\boldsymbol{k}T}{\delta}\right)\exp\left(-\delta/\boldsymbol{k}T\right)\right]$$

for chrome alum.

In applying these formulae we have used the values of δ derived from the entropy curves. The susceptibility χ_0 so calculated, in which allowance is made for the crystalline field, but not for magnetic interaction, may be considered as the susceptibility at infinite magnetic dilution. In Onsager's theory the magnetic interaction is considered to give rise to a local field. The calculation is purely classical, and takes no account of crystal structure, but it has the advantage of yielding an exact formula. In Van Vleck's theory the contribution of magnetic interaction to the partition function is calculated, and its influence on the magnetic susceptibility is then expressed as a power series in $1/T$, of which, unfortunately, we are able to use only the first term. Onsager's formula is

$$\chi=\chi_0\left[\frac{3}{4}-\frac{3}{16\pi\chi_0}+\left(\frac{9}{16}+\frac{3}{32\pi\chi_0}+\frac{9}{256\pi^2\chi_0^2}\right)^{\frac{1}{2}}\right].$$

The first term of Van Vleck's formula gives

$$\chi=\chi_0\left(1-\frac{4\pi}{3}\chi_0+12\eta\chi_0^2\right),$$

where

$$\eta=f\left(1+\frac{3}{8J(J+1)}\right).$$

Here f is again the lattice structure factor (1·20 for the alums, 1·4 for the manganese salt). T^* then follows from either of the formulae for χ by the equation

$$T^*=(\tau/3)(1/\chi+4\pi/3).$$

It will be seen from Figure 5 that either formula fits the experimental results on chrome alum well, over the range of temperatures investigated. The experiments on iron alum and manganous ammonium sulphate (Figures 3 and 4) show, however, that both theories fail at temperatures near the susceptibility maximum (minimum of T^*). The Onsager formula predicts no maximum, while the Van Vleck formula, though it predicts a susceptibility maximum, cannot really be considered to be valid at these temperatures owing to the very slow convergence of the power series in $1/T$ on which it is based. Moreover, neither formula predicts the sudden increase of susceptibility to the maximum value, nor the existence of a Curie point.

§5. THE CURIE POINT

The chief interest in the results obtained at the lowest temperatures is the appearance of a Curie point, below which remanence occurs, at a temperature of 0·042° K. in the case of iron ammonium alum, and 0·15° K. in the case of manganous ammonium sulphate. (Both of these temperatures are very near to the values at which the susceptibility maxima occur.) Of the different methods of calculating the magnetic interaction, only the Lorentz method gives a value sufficiently large to produce spontaneous magnetization. Onsager has shown that this method is inexact, but it is useful in that it gives an upper limit for the temperature at which dipole interaction can produce ferromagnetism. According to this theory the Curie point will depend on the shape of the specimen, an infinitely long specimen giving the highest Curie point, while no Curie point occurs for a sphere because of the neutralization of the demagnetizing and interaction fields. This dependence of the Curie point on the shape of the specimen has not been tested experimentally.

The specimens used in the experiments were ellipsoids with axial ratios of 3:1 and 4:1. Following the method of Debye (1938), the Curie points have been computed, using the values of δ already calculated. For iron ammonium alum, the calculated Curie point is 0·032° K. for an infinite cylinder, and 0·016° K. for a 3:1 ellipsoid. The observed Curie point of a 3:1 ellipsoid is 0·042° K. For manganous ammonium sulphate the calculated Curie point is 0·067° K. for an infinite cylinder, and 0·050° K. for a 4:1 ellipsoid. The observed Curie point for a 4:1 ellipsoid is 0·15° K. Although it must be emphasized that the determinations of the absolute temperatures are particularly difficult in the neighbourhood of the Curie points, it will be seen that the experimental values of the Curie points are decidedly higher than the calculated values, and in fact exceed the calculated values for infinite cylinders, which are the highest given by any theory of magnetic dipole–dipole interaction. It is clear that though the effect of the crystalline field is successfully treated by the theory, there are further interaction effects which have still to be explained.

ACKNOWLEDGMENTS

The author wishes to thank Professor F. Simon, Dr. B. Bleaney, Dr. R. A. Hull and Dr. N. Kurti for their permission to use many unpublished experimental results, and also for many helpful discussions. The measurements on iron alum were made by Dr. Kurti, Professor Simon and Dr. Squire, partly at the Laboratoire du Grand Electroaimant, Bellevue, near Paris, and partly at the Clarendon Laboratory. Those on chrome alum were made by Dr. Bleaney,

and those on manganous ammonium sulphate by Dr. Hull and the writer. Although they were all obtained some years ago, the results have not previously been published, as it was desired to make further measurements in the neighbourhood of the Curie points. It is felt, however, that this review, which was first given as a lecture to the Physical Society Conference in Cambridge in 1946, may be of value in the meantime.

REFERENCES

COOKE, A. H., and HULL, R. A., 1942, *Proc. Roy. Soc.* A, **181**, 83.
DEBYE, P., 1938, *Ann. Phys., Lpz.*, **32**, 85.
HEBB, M. H., and PURCELL, E. M., 1937, *J. Chem. Phys.*, **5**, 338.
HULL, J. R., and HULL, R. A., 1941, *J. Chem. Phys.*, **9**, 465.
HULL, R. A., 1947, *Report of Cambridge Conference on Low Temperatures* (London : Physical Society), p. 72.
KEESOM, W. H., 1934, *J. Phys. Radium*, **5**, 373.
KURTI, N., and SIMON, F., 1935 a, *Proc. Roy. Soc.* A, **149**, 161; 1935 b, *Ibid.*, **152**, 21; 1938, *Phil. Mag.*, **26**, 849.
LORENTZ, H. A., 1916, *The Theory of Electrons* (Leipzig : Teubner).
ONSAGER, L., 1936, *J. Amer. Chem. Soc.*, **58**, 1486.
PENNEY, W. G., and SCHLAPP, R., 1932, *Phys. Rev.*, **41**, 194.
SCHLAPP, R., and PENNEY, W. G., 1932, *Phys. Rev.*, **42**, 666.
VAN VLECK, J. H., 1937, *J. Chem. Phys.*, **5**, 320.
VAN VLECK, J. H., and PENNEY, W. G., 1934, *Phil. Mag.*, **17**, 961.

A Theory of Reflectivity and Emissivity

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ABSTRACT. A general theory of the reflectivity and emissivity of materials is derived in a conveniently tractable form. As an example of the power of the method, it is applied to the Drude-Zener treatment of the optical properties of an ideal metal containing perfectly free electrons only. On this basis it is shown that there should exist a wide region of constant reflectivity and emissivity in the near infra-red. Krönig's theory is shown to be an approximation to the exact expression. The temperature coefficient of reflectivity is investigated and it is seen, on certain general assumptions, that it cannot change sign except in the neighbourhood of a wavelength at which the metal becomes transparent.

Disagreement with experimental observations indicates that the Drude-Zener treatment must be modified, either by a fundamental change in the dispersion formula or, more probably, by assuming that the influence of the bound electron contribution to the optical behaviour of a metal is greater than had been supposed.

§1. INTRODUCTION

MANY experimental data on the reflectivity and emissivity of various materials over wide ranges of temperature and of wavelength are available. There is, however, considerable difficulty in interpreting this information theoretically except in the few special cases for which simple approximations can take the place of the more complex expressions derivable from the usual theory. This hindrance has left obscure many controversial points that are of considerable interest.

The theory of reflectivity and emissivity (which are related by Kirchhoff's Law, $E+R=1$) must be derived from that of the optical constants of materials. This in turn is connected by electromagnetic theory with the study of the dielectric constants and polarizabilities of the media. Quantum-mechanical considerations must be employed to give the dispersion laws for these quantities.

As might be expected from the nature of the steps involved, this chain of reasoning results, for example, in a quite intractable expression for the variation of reflectivity with wavelength, even for the much-simplified model of a metal containing only perfectly free electrons. Such instances of cumbrous expressions have been given by Richardson (1916) and by Weil (1948). The unwieldy structure prohibits physical interpretation and application to experimental data.

An effort is made here to simplify this mathematical machinery so that it may be used to obtain a consistent account of the optical properties of materials. The theory is presented in terms of a complex index of refraction, $N=n-ik$. This enables the method to be applied to the important problems arising in the study of absorbing media, in particular, metals.

§ 2. GENERAL THEORY

Dispersion theory is stated in terms of the polarizability of the medium, for it is this quantity which enters into the solutions of the differential equations governing the dispersing oscillators. Classical electromagnetic theory shows that $N^2=\epsilon=1+4\pi\alpha$, where ϵ is the complex dielectric constant and α is the complex polarizability of the medium. For convenience we shall write

$$N^2-1=4\pi\alpha=1/(X+iY). \qquad \ldots\ldots(1)$$

Now optical theory gives the well-known result that for normal incidence

$$R=1-E=\left|\frac{N-1}{N+1}\right|^2. \qquad \ldots\ldots(2)$$

We may therefore put $(N-1)/(N+1)=R^{\frac{1}{2}}e^{i\theta}$. Substituting this and (1) in the identity

$$\tfrac{1}{2}[(N-1)/(N+1)+(N+1)/(N-1)]=1+2/(N^2-1), \qquad \ldots\ldots(3)$$

we obtain on equating the real and the imaginary portions

$$\tfrac{1}{2}(R^{\frac{1}{2}}+R^{-\frac{1}{2}})\cos\theta=1+2X; \qquad \tfrac{1}{2}(R^{\frac{1}{2}}-R^{-\frac{1}{2}})\sin\theta=2Y. \qquad \ldots\ldots(4)$$

If we eliminate θ and set $\frac{1}{4}(R^{\frac{1}{2}}-R^{-\frac{1}{2}})^2=E^2/4R=y$ we obtain the equation

$$y^2-4y(Y^2+X^2+X)-4Y^2=0. \qquad \ldots\ldots(5)$$

This must always have two real roots of opposite sign. Since however R must be positive, the negative root is to be rejected. There is therefore a unique solution that can be written explicitly as

$$\tfrac{1}{2}y=E^2/8R=T+(T^2+Y^2)^{\frac{1}{2}}, \qquad \ldots\ldots(6)$$

where $T=Y^2+X^2+X$.

The simple result (6) is completely general and independent of the particular dispersion formula assumed for the medium. It would appear that the quantity E^2/R is more useful in reflectivity theory than either E or R.

§ 3. METAL CONTAINING FREE ELECTRONS ONLY

Modern theories agree that the polarizability of a model metal containing perfectly free electrons only can be represented by

$$-1/4\pi\alpha = (\lambda_0/\lambda)^2 + i(\lambda_\tau/\lambda), \qquad \ldots\ldots(7)$$

where $\lambda_0^2 = \pi mc^2/f_0e^2$ and $\lambda_\tau = c/2\sigma$, while f_0 is the number of free electrons per unit volume and σ is the electrical conductivity for steady currents. On the approximate basis of one free electron per atom, λ_0 is of the order of 0·1 μ, while λ_τ is of the order of 0·001 μ for most metals. We may assume without great danger that λ_τ/λ_0 is small compared with unity.

Substituting the dispersion equation (7) in (1) the use of the above method becomes apparent, for by aid of the artifice the two arbitrary constants are separated, and we find

$$X = -(\lambda_0/\lambda)^2; \qquad Y = -(\lambda_\tau/\lambda). \qquad \ldots\ldots(8)$$

A simple and compact equation for E^2/R may be obtained by substituting the above values in (6). For the present purposes, however, we substitute in (5) and obtain

$$y^2 - 4y[(\lambda_0/\lambda)^4 - (\lambda_0/\lambda)^2 + (\lambda_\tau/\lambda)^2] - 4(\lambda_\tau/\lambda)^2 = 0. \qquad \ldots\ldots(9)$$

From this exact equation it is seen that optical behaviour of such an ideal metal falls naturally into three main regions of the spectrum. In each of these regions useful approximations may be made.

(i) *Region of Transparency*, $\lambda \ll \lambda_0$

For very short wavelengths it is seen from (9) that $y = 4(\lambda_0/\lambda)^4$; under these conditions $n = 1 - \frac{1}{2}(\lambda/\lambda_0)^2$ and $k = \frac{1}{2}(\lambda_\tau/\lambda_0)(\lambda/\lambda_0)^3$, whence $R = \frac{1}{16}(\lambda/\lambda_0)^4$.

(ii) *Far Infra-red Region*, $\lambda \gg \lambda_0^2/\lambda_\tau$

For very long wavelengths the middle term of (9) may be neglected, and $y = 2(\lambda_\tau/\lambda)$. Expressed in the more familiar form, $E = 2(\boldsymbol{c}/\sigma\lambda)^{\frac{1}{2}}$, this is seen to be the Hagen-Rubens (or Drude) relation, the validity of which has been the subject of many experimental investigations. Fair agreement with the theory has been found.

(iii) *Intermediate Region*, $\lambda_0 \ll \lambda \ll \lambda_0^2/\lambda_\tau$

As in the transparent region (i), we neglect the term independent of y in (9) and obtain the negative solution $y = -4(\lambda_0/\lambda)^2$. Since the product of the two roots is $-4(\lambda_\tau/\lambda)^2$, the positive root is $y = (\lambda_\tau/\lambda_0)^2 = \text{constant}$. The constancy of y in this region is checked by noting that $k = (\lambda/\lambda_0)$ and $n = \frac{1}{2}(\lambda_\tau/\lambda_0)(\lambda/\lambda_0)^2$, giving $E = 2(\lambda_\tau/\lambda_0)$.

(iv) *Transition Regions*

Both the transition regions are rather abrupt. At the upper end we find that for $\lambda = 2\lambda_0^2/\lambda_\tau$ the approximations from above and from below both yield $y = (\lambda_\tau/\lambda_0)^2$, whereas the true value is only 0·618 of this. At the lower end the whole transition extends approximately from $\lambda_0/2$ to $3\lambda_0/2$, although there is a central region of width λ_τ centred about λ_0 over which the main portion of the transition occurs.

The most important result of this section is the appearance of the intermediate region (iii) for the commonly adopted model of an ideal metal. This shows that there exists a wide range of wavelengths, extending from the visible region up to the lower limit of validity of the Hagen-Rubens relationship, over which the emissivity is, to a good approximation, constant and equal to $2(\lambda_\tau/\lambda_0)$.

§ 4. TEMPERATURE VARIATION

The partial differentiation of (5) gives

$$\frac{\partial y}{\partial X}=\frac{4y^{\frac{3}{2}}(2X+1)}{y^2+4Y^2}; \qquad \frac{\partial y}{\partial Y}=\frac{8yY(y+1)}{y^2+4Y^2}. \qquad \ldots\ldots(10)$$

It will be observed that except for zero, negative, and infinite values of y these derivatives can only become zero if $X=-\frac{1}{2}$ for the first and $Y=0$ for the second. This latter case implies that N is real, and, therefore, that the medium is completely transparent. Now the existence of a stationary point for E or R necessitates that y also be a stationary value. Hence if we assume that Y, but not X, is dependent on the temperature t then dy/dt can never be zero for wavelengths between zero and infinity, except at a point for which the medium is transparent. This point occurs only at very short ultra-violet wavelengths and, therefore, beyond this the possibility is precluded of the temperature effect ever being in the direction opposite to that predicted from the Hagen-Rubens relation. In the intermediate region of constant emissivity and reflectivity y is proportional to λ_τ^2, whereas in the long-wave region it is proportional to λ_τ; we should therefore expect the temperature dependence to *increase* for decreasing wavelength up to the limit of the transparent region where the variation with temperature becomes suddenly negligible. This is in contradiction to many experimental results for real metals, and we see therefore that more complex assumptions must be made. Either it must be assumed that X (and hence λ_0) is temperature dependent, or the dispersion formula must be radically altered. The variation with temperature of λ_0 has usually been supposed small in comparison with that of electrical conductivity (i.e. λ_τ). It follows then that the phenomenon of an X-point (see Price 1947) and the existence of the intermediate region of constant reflectivity demand more experimental attention and perhaps a revision of the present dispersion theory. The contribution of the bound electrons has been neglected in the above treatment, as indeed has almost always been the case in the theoretical investigation of experimental results on the optical properties of metals. Kent (1919) considered the metal as being a combination of a free electron portion with a dielectric portion of constant refractive index, and obtained striking agreement between theory and experiment for a number of molten metals. Incorporation of the bound-electron contribution into (7) might therefore make the difference required to remove the discrepancies stated above. This treatment can only be approximate, and hence precise formulation remains conjectural.

§ 5. COMPARISON WITH THE THEORY OF KRÖNIG

Equation (6) may be directly compared with a similar one obtained by Krönig (1929, 1931) in his consideration of the optics of a metal containing one class of (free) electrons only. His result, stated in terms of frequency instead of wavelength, is

$$E^2=4(\nu/\sigma)[(u^2+1)^{\frac{1}{2}}-u], \qquad \ldots\ldots(11)$$

where σ is the electrical conductivity of the surface layers of the metal and $u=\nu\sigma/\beta$, where β is the half-breadth of the resonance line, $\nu=0$, in the equivalent spectrum of the electrons. Comparing the above approximation with the exact expression of (6), we note that Krönig has taken R to be sensibly equal to unity, so obtaining E^2 instead of E^2/R on the left-hand side. He also assumes that $Y^2=(\nu/2\sigma)^2$ and

that $T=-\nu^2/2\beta$. The first of these expressions follows from (8), the second is the first term of the exact form

$$T=-(\lambda_0/\lambda)^2+(\lambda_0/\lambda)^4+(\lambda_r/\lambda)^4, \qquad \ldots\ldots(12)$$

provided that we set $c/\beta=\lambda_0^2/\lambda_r$. It follows that the half-breadth of the line of

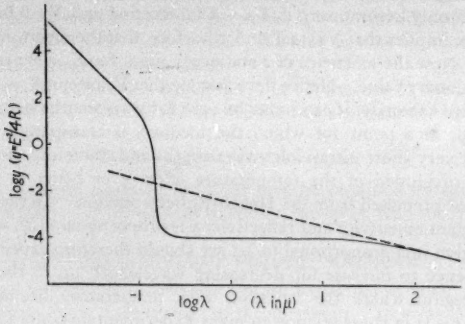

Figure 1. The variation of E^2/R with wavelength for an ideal metal having constants $\lambda_0=0{\cdot}16\,\mu$ and $\lambda_r=0{\cdot}0016\,\mu$. The dotted line shows the Hagen-Rubens approximation.

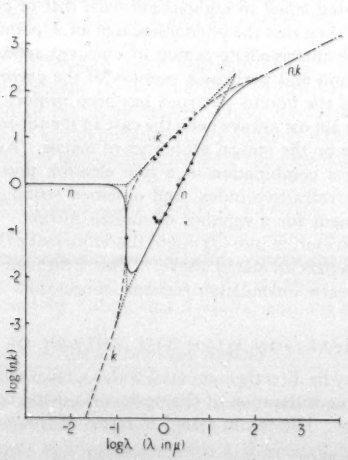

Figure 2. The variation of the optical constants n and k with wavelength for the same ideal metal. The dots indicate the experimental observations on silver at room temperature obtained by Försterling and Fréedericksz (1913).

zero frequency is simply half the wavelength that marks the lower limit of validity of the Hagen-Rubens relationship.

From the nature of the approximation to (12) it is seen that the third term may safely be neglected in comparison with the first, since (λ_τ/λ_0) is very small for most metals. The second term, however, can only be neglected for wavelengths much larger than λ_0.

§6. SPECTRAL VARIATION OF $E^2/4R$, n, AND k

The properties of the intermediate region found by applying the general exact theory to the case of a model metal act as a link between the already well-known properties in the far infra-red region and in the region of short optical wavelengths. Simple graphs of the wavelength variation of the optical properties of an ideal metal are obtainable by the use of the variable $y = E^2/4R$ and the adoption of logarithmic scales. In Figures 1 and 2 the three main spectral regions are clearly shown to have excellent linear approximations. The form of the transition between the region is also demonstrated; the information may be used to complete the account given by Seitz (1940).

It must be emphasized that equation (6) is perfectly general; all arbitrary assumptions are introduced by (7), and it is this part of the argument that must be modified if the resulting deductions of the presence of a region of constant reflectivity and of the absence of an X-point are found to be in conflict with experiment.

The general method can be applied equally well to the case of a dielectric containing one class of (bound) electrons only. The modification may be effected by a change in form of X only, Y remaining substantially unaltered. We then require $X = \gamma^2 - (\lambda_0/\lambda)^2$, where γ is an arbitrary constant depending on the resonance wavelength λ_0/γ of the class of electrons responsible for dispersion. Application to the cases of media containing more than one class of electrons is more difficult since α, but not $1/\alpha$, is additive for the different classes.

REFERENCES

FÖRSTERLING, K., and FRÉEDERICKSZ, V., 1913, *Ann. Phys., Lpz.*, **40**, 201.

KENT, C. V., 1919, *Phys. Rev.*, **14**, 459.

KRÖNIG, R. DE L., 1929, *Proc. Roy. Soc.* A, **124**, 409; 1931, *Ibid.*, **133**, 255.

PRICE, D. J., 1947, *Proc. Phys. Soc.*, **59**, 131.

RICHARDSON, O. W., 1916, *The Electron Theory of Matter* (Cambridge: University Press), p. 179.

SEITZ, F., 1940, *Modern Theory of Solids* (New York and London: McGraw-Hill), p. 639.

WEIL, R. A., 1948, *Proc. Phys. Soc.*, **60**, 8.

The Electrical Conductivity of Germanium *

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ABSTRACT. Measurements of the electrical conductivity of germanium are described. The results can be explained by the theoretical calculations of Shifrin (1944). An account is given of the relevant part of Shifrin's work. The experimental results are used to deduce the concentration of impurity centres and of thermally excited electrons and the position of the impurity levels.

§ 1. INTRODUCTION

THE object of this paper is to describe and explain some measurements of the electrical conductivity of germanium made by the writer.

The experimental results are of similar form to those described by Lark-Horovitz and his co-workers (1946), but they are explained here by means of a simpler theoretical model than that used by Lark-Horovitz. The hypothesis of Rutherford scattering by impurity centres has not been used to explain the observed fall in conductivity at liquid air temperatures. A complete explanation of the observed results has been given by Shifrin (1944); but it has been felt necessary to present here the relevant parts of Shifrin's work both for completeness and clarity. It has also been possible to deduce the concentration of impurity centres and the position of the impurity levels. The concentration of electrons in the conduction band may be found at low temperatures.

§ 2. METHOD OF MEASUREMENT AND EXPERIMENTAL RESULTS

Germanium specimens were cast in the form of rods, about 1–2 cm. long and about 0·5 cm. diameter. Their resistances were measured by a potentiometer method. A current of the order of 0·5 amp. was passed through the specimen and the potential difference between two probes was measured with a potentiometer. Chromel-alumel thermocouples were used for the potential leads, so that the temperature of the specimen could also be determined. To eliminate thermoelectric effects the current through the specimen was reversed and the mean was taken of the two results obtained.

Measurements were made over the temperature range 90° K. to 950° K. and on three different specimens. These had all been cast in silica tubes, No. 1 in hydrogen and Nos. 2 and 3 *in vacuo*.

Observations of the sign of the thermoelectric power showed that for all specimens the charge carriers were negative.

The results obtained are shown graphically. In Figure 1 the logarithm of σ the conductivity is plotted against the reciprocal of the absolute temperature T.

The results for the different specimens have the following features in common. Starting at liquid air temperature, $\log\sigma$ increases linearly with $1/T$ until at a temperature around 200–300° K. $\log\sigma$ passes through a maximum. $\mathrm{Log}\,\sigma$ then

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decreases until at $T \simeq 500°$ K. it passes through a minimum. Thereafter $\log\sigma$ increases very rapidly along a straight line which is the same for all specimens. The results are thus of the same form as reported by Lark-Horovitz.

In Figure 2 σ is plotted against T, using the same results as are used in Figure 1. It is seen that in the region 300–500° K. σ falls off approximately linearly with T for all specimens. This result agrees with the predictions of Shifrin.

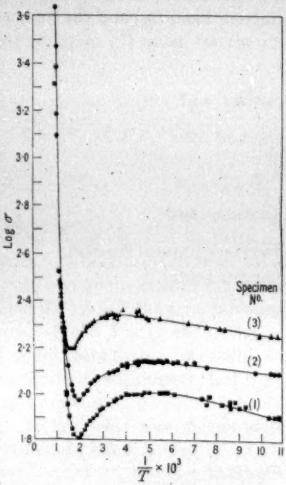

Figure 1.

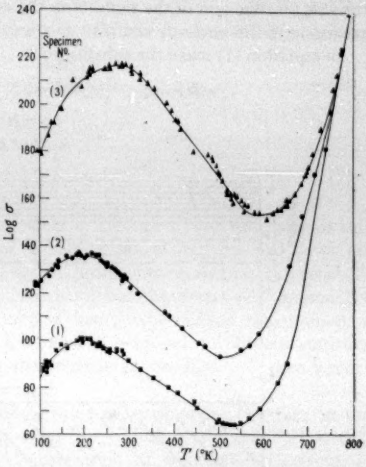

Figure 2.

§ 3. SOME PROPERTIES OF IMPURITY SEMICONDUCTORS CONTAINING A LARGE CONCENTRATION OF IMPURITY CENTRES

The system of energy levels which will be used to explain the behaviour of germanium is shown in Figure 3. n_b impurity centres per unit volume are situated at a distance ϵ' below the bottom of the conduction band. At a temperature T, n_f electrons are excited into the conduction band. It is assumed that at temperatures below 500° K. all the electrons come from the impurity centres. It will also be assumed that ϵ' is of the same order as $\boldsymbol{k}T$ at room temperature (where $\boldsymbol{k}$ is Boltzmann's constant) and that the concentration of electrons in the conduction band is so high that Fermi–Dirac statistics must be used to discuss the problem.

Before using this model to calculate the conductivity some general properties of it will be discussed. Most of the conclusions reached were first pointed out by Shifrin (1944) but as detailed accounts of his work are not readily available, it seems desirable to repeat some of his work.

In this section the chemical potential and the concentration of free electrons will be calculated as a function of temperature, and the conditions for which it is necessary to use Fermi statistics will be discussed.

Seitz (1940) has shown that the chemical potential ϵ^* is found by solving this equation:

$$n_b = n_b \frac{1}{1+\exp\left[-(\epsilon'-\epsilon^*)/\boldsymbol{k}T\right]} + \frac{4\pi(2m^*)^{\frac{3}{2}}}{\boldsymbol{h}^3}\int_0^\infty \frac{\epsilon^{\frac{1}{2}}\,d\epsilon}{1+\exp\left[(\epsilon-\epsilon^*)/\boldsymbol{k}T\right]} \quad \ldots\ldots(1)$$

where $\boldsymbol{h}$ is Planck's constant, m^* the effective mass of an electron in the conduction band, and ϵ energy, the zero of energy being taken at the bottom of the conduction band. This equation is derived by equating the total number of impurity electrons (n_b) to the sum of the number in the conduction band (n_f) and the number remaining in the impurity centres.

In equation (1) make the substitutions

$$\epsilon/\boldsymbol{k}T=\eta, \qquad \epsilon'/\boldsymbol{k}T=\eta', \qquad \epsilon^*/\boldsymbol{k}T=\eta^*$$

and $$A=\frac{\eta_b\boldsymbol{h}^3}{4\pi(2m^*\boldsymbol{k}T)^{\frac{3}{2}}}.$$

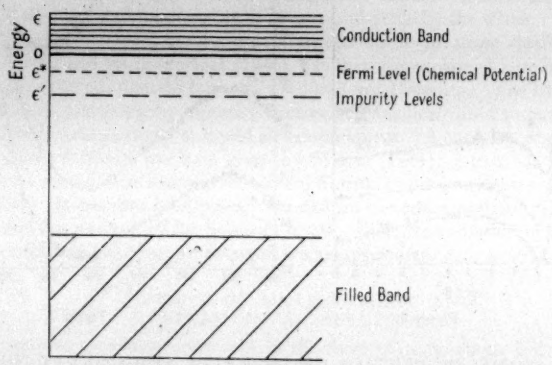

Figure 3.

The equation can then be written

$$\frac{A}{1+\exp(\eta'+\eta^*)} = \int_0^\infty \frac{\eta^{\frac{1}{2}}\,d\eta}{1+\exp(\eta-\eta^*)}; \quad \ldots\ldots(2)$$

also, from the second term on the right hand side of (1), the following expression for n_f is obtained

$$n_f = \frac{4\pi(2m^*\boldsymbol{k}T)^{\frac{3}{2}}}{\boldsymbol{h}^3}\int_0^\infty \frac{\eta^{\frac{1}{2}}\,d\eta}{1+\exp(\eta-\eta^*)}. \quad \ldots\ldots(3)$$

Equation (2) was first derived by Shifrin and a similar expression has been derived by Busch (1946). In order to use equations (2) and (3) without making approximations, the integral

$$F_{\frac{1}{2}}(\eta^*) = \int_0^\infty \frac{\eta^{\frac{1}{2}}\,d\eta}{1+\exp(\eta-\eta^*)}, \quad \ldots\ldots(4)$$

must be evaluated. This integral cannot be evaluated directly, but it has been tabulated accurately by McDougall and Stoner (1938).

Neither Shifrin nor Busch seem to be aware of this table because Shifrin tabulates a few values of $\frac{1}{2}F_{\frac{1}{2}}(\eta^*)$, some of which are as much as 3% in error and Busch uses Shifrin's values in his paper.

With $F_{\frac{1}{2}}(\eta^*)$ known, it is possible to determine η^* by solving equation (2) numerically. By this means η^* can be found as accurately as desired in all cases. We wish to consider when this procedure is necessary. When classical statistics can be applied $F_{\frac{1}{2}}(\eta^*)$ reduces to

$$F_{\frac{1}{2}}(\eta^*) = \tfrac{1}{2}\sqrt{\pi}\exp\eta^* \qquad \ldots\ldots(5)$$

and equation (3) gives for the chemical potential the semi-classical expression

$$\eta^* = \ln\frac{n_f h^3}{2(2\pi m^* kT)^{\frac{3}{2}}}. \qquad \ldots\ldots(6)$$

For $\eta^* < 0$, $F_{\frac{1}{2}}(\eta^*)$ can be represented by the series

$$F_{\frac{1}{2}}(\eta^*) = \frac{\sqrt{\pi}}{2}\exp\eta^* \sum_{n=0}^{\infty}(-1)^n \frac{\exp n\eta^*}{(1+n)^{\frac{3}{2}}}, \qquad \ldots\ldots(7)$$

so that using classical statistics is equivalent to taking only the first term of this series. Therefore the error involved in using classical statistics will be of the same order as the ratio of the first and second terms of the series, i.e. as $\exp\eta^*/2^{\frac{3}{2}}$. Thus if $\eta^* = -2$, the error is about 5%, and it will become greater as η^* approaches zero. Since 5% is about the accuracy of many experimental measurements, it would seem that in cases where η^* lies between $-\infty$ and -2 classical statistics may be used with safety, but if η^* is algebraically greater than -2, then a more exact method of calculation should be used.

To see what this means in terms of the concentration of electrons in the conduction band, put $\eta^* = -2$ in equation (3). Then $n_f = 6{\cdot}1 \times 10^{14} T^{\frac{3}{2}}$ so that for $T = 290°$ K., $n_f = 3 \times 10^{18}$ cm^{-3}. Concentration of conduction electrons of this order are found in specimens of silicon and germanium (Torrey and Whitmer 1948, p. 53 ff), so that the use of classical statistics to discuss these substances does not seem to be adequate.

To illustrate how η^* and n_f will vary with temperature, the numerical results given in Tables 1 and 2 were calculated. These were calculated for a semiconductor with impurity levels 0·03 ev. below the bottom of the conduction band as this is about the value found in germanium.

Table 1. Variation of Reduced Chemical Potential η^* and Concentration of Conduction Electrons n_f with the Concentration of Impurity Centres n_b.

($T = 290°$ K. Depth of impurity levels below the conduction band, $\epsilon' = 0{\cdot}03$ ev.)

n_b (cm^{-3})	10^{18}	10^{19}	10^{20}	10^{21}	10^{22}
n_b (as atomic % in Ge)	0·0022	0·022	0·22	2·2	22·0
η^*	−3·3	−1·4	+0·15	+1·63	+3·26
n_f (cm^{-3})	$8{\cdot}71 \times 10^{17}$	$5{\cdot}44 \times 10^{18}$	$2{\cdot}06 \times 10^{19}$	$5{\cdot}53 \times 10^{19}$	$1{\cdot}19 \times 10^{20}$

Table 1 shows how η^* and n_f vary at room temperature with n_b, the concentration of impurity centres. It is seen that if $n_b \geqslant 10^{19}$ cm^{-3} the more exact method of calculation should be used. The values of η^* were found by solving equation (2) and n_f was then found using (3). It will be seen that while n_b varies by four powers of ten, n_f varies only by about two, so that the substance is becoming appreciably

degenerate. For germanium, an impurity concentration of 10^{19} cm^{-3} corresponds to 0·022 atomic % of impurities. Reference to Torrey and Whitmer (1948) shows that the concentration of impurities used in germanium melts prepared for crystal valves is often considerably greater than this. Therefore these calculations may explain the observation reported by Lark-Horovitz *et al.* (1946) that the number of free electrons at room temperature deduced from Hall effect measurements is appreciably less than the number which would be expected if all the impurity atoms were ionized. This is probably not the complete explanation, because the results of measurements made down to liquid helium temperatures suggest that the simple model is not adequate to account for the behaviour of specimens containing very large concentrations of impurities (Pearson and Shockley 1947).

Table 2 shows how η^* and n_f vary with temperature for two values of n_b.

Table 2. Variation of η^* and n_f with T for $\epsilon' = 0{\cdot}03$ ev.

	(*a*) $n_b = 10^{18}$ cm^{-3}.			(*b*) $n_b = 10^{20}$ cm^{-3}.	
T (° K.)	η^*	n_f (cm^{-3})	T (° K.)	η^*	n_f (cm^{-3})
10	−16·46	$1{\cdot}09\times10^{10}$	10	−14·15	$1{\cdot}10\times10^{11}$
50	−3·75	$4{\cdot}03\times10^{16}$	50	−1·41	$3{\cdot}86\times10^{17}$
100	−2·71	$3{\cdot}15\times10^{17}$	100	−0·12	$3{\cdot}37\times10^{18}$
150	−2·72	$6{\cdot}73\times10^{17}$	150	+0·16	$7{\cdot}71\times10^{18}$
200	−2·88	$7{\cdot}54\times10^{17}$	200	+0·25	$1{\cdot}27\times10^{19}$
250	−3·10	$8{\cdot}49\times10^{17}$	250	+0·19	$1{\cdot}70\times10^{19}$
290	−3·30	$8{\cdot}71\times10^{17}$	290	+0·15	$2{\cdot}06\times10^{19}$
350	−3·57	$8{\cdot}89\times10^{17}$	350	+0·07	$2{\cdot}57\times10^{19}$
400	−3·70	$9{\cdot}50\times10^{17}$	400	0·00	$2{\cdot}96\times10^{19}$

At very low temperatures η^* has a large negative value and n_f is very small. η^* increases rapidly as the temperature rises and n_f increases exponentially. Above a certain temperature, the rate of increase of η^* falls off, η^* passes through a maximum, which may be positive, and then starts to decrease. At high temperatures η^* will be negative. Although η^* exhibits a maximum, n_f increases steadily throughout.

Comparison of the two cases shown in Table 2 shows that when $n_b = 10^{18}$ cm^{-3} the behaviour can be calculated classically and at 400° K. 95% of the impurities are ionized, but with $n_b = 10^{20}$ there is some evidence of the electron gas becoming degenerate, so that at 400° K. only 30% of the impurities are ionized.

In both the cases described η^* passes through a maximum. It is possible to derive a relation between the maximum value of η^* and the temperature corresponding to it. This relation will be used in examining the experimental results, and a derivation of it will be given in the next section.

§ 4. DERIVATION OF THE RELATION BETWEEN THE MAXIMUM VALUE OF η^* AND TEMPERATURE

To derive the required expression, write the quantity A in equation (2) in the form:

$$A = a\eta'^{\frac{3}{2}} \qquad \text{where} \qquad a = n_b\boldsymbol{h}^3/4\pi(2m^*\epsilon')^{\frac{3}{2}} \qquad \text{and} \qquad \eta' = \epsilon'/\boldsymbol{k}T.$$

While a depends upon the properties of the semiconductor it is independent of temperature. Equation (2) can now be re-written,

$$\eta'^{\frac{3}{2}}a/F_{\frac{1}{2}}(\eta^*) = 1 + \exp(\eta' + \eta^*). \qquad \ldots\ldots(8)$$

Consider some given value of η^*. Then equation (8) can be regarded as an equation to determine the value of η' (and hence T, since $\eta' = \epsilon'/kT$) corresponding to the value taken for η^*. The nature of the roots of (8) can be seen by considering the curves

$$\left.\begin{aligned} y_1 &= f_1(\eta') = \eta'^{\frac{3}{2}} a / F_{\frac{1}{2}}(\eta^*) \\ y_2 &= f_2(\eta') = 1 + \exp(\eta' + \eta^*) \end{aligned}\right\} . \qquad \ldots\ldots (9)$$

It is seen that $y_2 > y_1$, both for $\eta' \sim 0$ and for $\eta' \to \infty$, but for intermediate values of η', y_2 may be less than y_1 if the value taken for η^* is not too large; for very large values of η^* the curves will not intersect. Thus, in general, equation (8) will have either two real roots or none. Although, mathematically, two real roots may exist, they are not always both physically realizable, for there is always a maximum temperature at which a substance behaves as an impurity semiconductor; either it will melt or decompose or the number of electrons coming

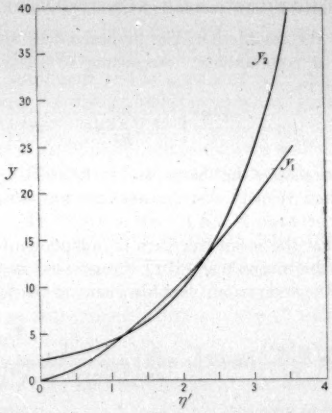

Figure 4.

Equation (9) $y_1 = \dfrac{a}{F_{\frac{1}{2}}(\eta^*)}\,\eta'^{\frac{3}{2}}$; $\quad y_2 = 1 + \exp(\eta^* + \eta')$

with $\epsilon' = 0{\cdot}03$ ev., $n_b = 10^{20}$ cm^3, $\eta^* = +0{\cdot}15$

giving $y_1 = 3{\cdot}70\,\eta'^{\frac{3}{2}}$; $\quad y_2 = 1 + \exp(0{\cdot}15 + \eta')$.

from the full band becomes appreciable. Hence the smaller root of (8) (which corresponds to the higher temperature) may not always be physically significant.

Figure 4 illustrates the behaviour of the curves of equation (9). It has been drawn for the case tabulated in Table 2(*b*), taking $\eta^* = 0{\cdot}15$. The two points of intersection correspond to temperatures of about 290° K. and 145° K.

While, as we have seen, there are in general two roots to equation (8), there will be a limiting case when the two curves (9) touch. This will give a double root corresponding to a stationary value of η^* with respect to T. For this case, we have

$$\frac{dy_1}{d\eta'} = \frac{dy_2}{d\eta'}, \qquad \text{i.e.} \qquad \frac{3}{2}\frac{a}{F_{\frac{1}{2}}(\eta^*)}\,\eta'^{\frac{1}{2}} = \exp(\eta' + \eta^*).$$

Remembering that this value of η' must satisfy (8), we obtain

$$\tfrac{2}{3}\eta'_{\max} = 1 + \exp(-\eta'_{\max} - \eta^*_{\max})$$

or

$$\eta^*_{\max} = -\ln(2\epsilon'/3\boldsymbol{k}T_{\max} - 1) - \epsilon'/\boldsymbol{k}T_{\max}. \qquad \ldots\ldots(10)$$

That this equation corresponds to a maximum value of η^* can be seen by considering the behaviour of the curves (9) as η^* is varied. The subscript max is attached to η' and T to indicate that they have the values for which η^* is a maximum.

The data in Table 2 illustrate how η^* can pass through a maximum, and it is seen that the maximum may be positive or negative and that as the concentration of impurities is increased, both $\eta^*_{\max}$ and $T_{\max}$ become larger.

The object of this section has been to derive equation (10). The application of this equation will be discussed later.

§ 5. DERIVATION OF AN EXPRESSION FOR THE ELECTRICAL CONDUCTIVITY AS A FUNCTION OF η^*

An expression for the conductivity can be derived by applying the Lorentz–Sommerfeld theory of conductivity. According to Seitz (1940, p. 189) the conductivity σ is given by,

$$\sigma = -\frac{4\pi\boldsymbol{e}^2}{3}\int_0^\infty V^3 l \frac{\partial f_n^0}{\partial x}\,dV \qquad \ldots\ldots(11)$$

where l is the mean free path, $\boldsymbol{e}$ the charge, and V the scalar speed of the electrons, and f_n^0 the unperturbed velocity distribution function, so that the number of electrons with speeds between V and $V+dV$ is $4\pi f_n^0 V^2\,dV$.

If it is assumed that the mean free path is independent of energy, so that l can be taken outside the integral, and if f_n^0 is expressed in terms of the Fermi–Dirac function, and the integration variable changed to the reduced energy η, then (11) becomes

$$\sigma = \frac{16\pi}{3}\left(\frac{m^*\boldsymbol{e}^2}{\boldsymbol{h}^3}\right)(l\boldsymbol{k}T)\int_0^\infty \eta \frac{d}{d\eta}\left(\frac{1}{1+\exp(\eta-\eta^*)}\right)d\eta. \qquad \ldots\ldots(12)$$

This can be integrated to give

$$\sigma = \frac{16\pi}{3}\left(\frac{m^*\boldsymbol{e}^2}{\boldsymbol{h}^3}\right)(l\boldsymbol{k}T)\ln(1+\exp\eta^*). \qquad \ldots\ldots(13)$$

This expression should hold for all values of η^*, but if η^* is large and negative (i.e. the classical case), then the logarithm in (13) can be expanded. If only the first term is retained, the resulting expression for σ will be found to be identical with that derived by Seitz (1940) for a simple semiconductor.

If we now assume that the principal source of scattering which determines the mean free path is the interaction of electrons with the crystal lattice vibrations, then l will vary as $1/T$, as has been shown by Wilson (1936, p. 211), Sommerfeld and Bethe (1933, p. 560) and Seitz (1948).

We can now write $l = l_0/\boldsymbol{k}T$ and (13) becomes

$$\sigma = \sigma_0 \ln(1+\exp\eta^*) \qquad \ldots\ldots(14)$$

where $\sigma_0 = \dfrac{16\pi}{3}\left(\dfrac{m^*\boldsymbol{e}^2}{\boldsymbol{h}^3}\right)l_0$. Thus σ_0 is independent of temperature.

§6. DISCUSSION OF RESULTS

If the variation of η^* with T is known, then the variation of σ can be found at once. From our knowledge of the behaviour of η^* the behaviour of σ can be seen. Starting at very low temperatures, σ will increase exponentially since at low enough temperatures the simpler formula will apply. Thus a plot of $\log\sigma$ against $1/T$ will be a straight line at low temperatures. At higher temperatures η^* will pass through a maximum and then decrease. Since $\ln(1+\exp\eta^*)$ is a monotonic function of η^*, σ will be a maximum at the same temperature at which η^* is maximum. As the temperature is raised further, η^* will decrease and σ will decrease with it. Shifrin has stated that σ would be expected to decrease approximately linearly with temperature in this region.

Examination of the experimental results (Figures 1 and 2) shows that up to $T=500°$ K., the conductivity behaves as predicted by this simple model. At very low temperature, the exponential variation is obtained. The conductivity passes through a maximum at about 200° K. and thereafter decreases linearly with T. At about 500° K. the conductivity passes through a minimum and then begins to vary exponentially again. Thus above 500° K. the specimens no longer behave as impurity semiconductors. The conductivity at high temperatures has the same value for all specimens and is associated with the excitation of electrons from the filled band, so that at high temperatures germanium behaves as an intrinsic semiconductor.

Thus the model of an impurity semiconductor as discussed here explains the observed results. In the next section an attempt will be made to obtain quantitative data for the specimens measured.

§7. APPLICATION OF THEORY

The concentration of free electrons, impurity centres and the position of the impurity energy levels were calculated by applying the following arguments.

In the low temperature region where $\sigma \propto \exp\eta^*$ write $\eta^* = -K/T$ where K is a positive constant.

From the slope of the straight line of $\log\sigma$ plotted against $1/T$, the value of K is found. This is then used to calculate η^* and hence $\ln(1+\exp\eta^*)$ over the low temperature range.

Recalling the fundamental equation (14)

$$\sigma = \sigma_0 \ln(1+\exp\eta^*)$$

we can now use the measured values of σ to determine σ_0.

Since we are assuming σ_0 to be independent of temperature we can now find η^* at any temperature from the conductivity equation.

When η^* is known, the concentration of free electrons can at once be determined, for the equation (3) for n_f can be expressed

$$n_f = 5{\cdot}47\times10^{15}T^{\frac{3}{2}}F_{\frac{1}{2}}(\eta^*). \qquad \ldots\ldots(15)$$

The values for $F_{\frac{1}{2}}(\eta^*)$ were obtained from the table of McDougall and Stoner referred to above.

To determine the depth of the impurity centres below the bottom of the conduction band we use the maximum value of the conductivity σ_{max} to calculate η^*_{max}. This is then substituted in equation (10) and ϵ' is found by solving this equation, since T_{max} is the temperature of maximum conductivity.

Finally we can calculate the density of impurities. Equation (2) can be written

$$n_b = 5{\cdot}47 \times 10^{15}[1+\exp(\eta'+\eta^*)]T^{\frac{3}{2}}F_{\frac{1}{2}}(\eta^*). \qquad \ldots\ldots(16)$$

Since all the quantities on the right hand side are known n_b can be calculated. These calculations were applied to the three specimens measured and the results are given in Table 3.

Table 3.

Specimen	n_b (cm^{-3})	n_b (atomic %)	n_f (cm^{-3}) at 290° K.	ϵ'	σ_0 ($ohm^{-1}\,cm^{-1}$)
1	$4{\cdot}4\times10^{19}$	0·097	$1{\cdot}41\times10^{19}$	0·0297	174
2	$6{\cdot}4\times10^{19}$	0·141	$1{\cdot}61\times10^{19}$	0·0306	208
3	$8{\cdot}9\times10^{19}$	0·196	$1{\cdot}66\times10^{19}$	0·0383	350

The approximations involved in these calculations are of the order of 10%, and the accuracy of the experimental measurements is somewhat better than this.

§8. CONCLUSION

The experimental measurements of the electrical conductivity of germanium can be explained by using a simple model of a semiconductor and applying Shifrin's theory to it.

It is possible to determine the concentration of impurity centres, the position of their energy levels and the concentration of conduction electrons by means of this theory.

ACKNOWLEDGMENTS

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REFERENCES

Busch, G., 1946, *Helv. Phys. Acta*, **19**, 463.

Lark-Horovitz, K., Middleton, A. E., Miller, E. P., and Walerstein, I., 1946, *Phys. Rev.*, **69**, 258.

McDougall, J., and Stoner, E. C., 1938, *Phil. Trans. Roy. Soc.*, **237**, 67.

Pearson, G. L., and Shockley, W., 1947, *Phys. Rev.*, **71**, 141.

Seitz, F., 1940, *Modern Theory of Solids* (New York and London: McGraw-Hill); 1948, *Phys. Rev.*, **73**, 549.

Shifrin, K., 1944, *J. Phys. U.S.S.R.*, **8**, 242.

Sommerfeld, A., and Bethe, H. A., 1933, *Handbuch der Physik* XXIV/2 (Berlin: Springer).

Torrey, H. C., and Whitmer, C. A., 1948, *Crystal Rectifiers, Radiation Laboratory Series* No. 15, (New York and London: McGraw-Hill).

Wilson, A. H., 1936, *The Theory of Metals* (Cambridge: University Press).

Measurement of the Half-Life of ^{6}He

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Communicated by P. B. Moon; MS. received 9th December 1948

ABSTRACT. The half-life of ^{6}He is found to be $0{\cdot}823 \pm 0{\cdot}013$ sec. by a rotating wheel method. The statistical accuracy is considerably higher than that of previous measurements, though all results agree to within their limits of error.

§ 1. INTRODUCTION

Bjerge (1936) was the first to identify ^{6}He, which is formed when beryllium is bombarded by fast neutrons; by a continuous gas-flow method he estimated the half-life of the beta-activity to be of the order of 1 second. Later, Nahmias and Walen (1937) gave a value of $0{\cdot}80 \pm 0{\cdot}04$ sec.; in their method the sample was mounted on a "pendulum" swinging between a neutron source and a G–M counter, the process of alternate irradiation and recording of the activity being repeated many times. More recently two measurements have been made using neutrons generated by a cyclotron. Sommers and Sherr (1946) obtained a value of $0{\cdot}85 \pm 0{\cdot}05$ sec. using a discontinuous flow method, the activity being transported from the cyclotron by hydrogen under pressure through a pipe 120 feet long to the counting chamber. Cassels and Latham (1947) used a pulsed cyclotron in conjunction with electronic gating circuits and obtained a value $0{\cdot}87 \pm 0{\cdot}06$ sec.

§ 2. EXPERIMENTAL ARRANGEMENT

The apparatus used for this experiment was constructed by the author and Mr. R. S. Turgel for a search for short-lived isomers. It consisted essentially

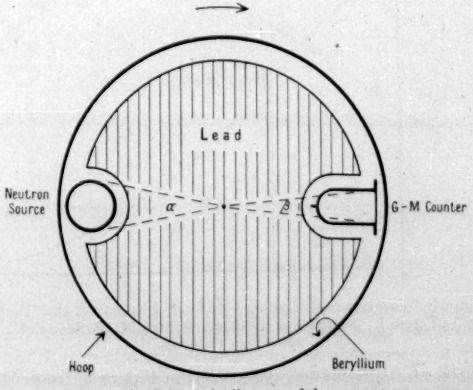

Figure 1. Schematic diagram of the apparatus.

of a steel hoop 45 cm. in diameter carrying 50 gm. of powdered beryllium metal held in position by cellophane tape in a shallow groove running round the interior

surface. The hoop was rotated continuously past a Ra–Be neutron source containing 230 mg. Ra and then in front of an end-on type β-counter. The arrangement is shown schematically in Figure 1. The whole apparatus was heavily shielded with lead to screen the G–M counter from the γ-radiation emitted by the neutron source. This principle has been used previously by Nahmias and Walen (1937) to measure the half-life of short-lived β-activities.

When the hoop has been rotating at a constant speed for some time, the counter will record a constant counting rate due to the activity produced in the beryllium when irradiated with fast neutrons, allowing continuous counting over an extended period of time.

§3. THEORY OF THE METHOD

As shown by Nahmias and Walen (1937), the equilibrium counting rate C is given by

$$C = K\frac{\omega}{\lambda}\{1-\exp(-\lambda\alpha/\omega)\}\{1-\exp(-\lambda\beta/\omega)\}\frac{\exp(-\lambda\phi/\omega)}{1-\exp(-2\lambda\pi/\omega)}, \quad \ldots\ldots(1)$$

where

λ = transformation constant of the activity,
ω = angular velocity of the hoop,
α = effective angular width of the source,
β = effective angular width of the counter,
$\phi = \pi - (\alpha+\beta)/2$,
K = constant.

If $\lambda\alpha \ll \omega$, $\lambda\beta \ll \omega$ and $\phi \simeq \pi$, this may be simplified to

$$C = C_0 x/\sinh x, \quad \ldots\ldots(2)$$

where $x = \lambda\pi/\omega = 0{\cdot}347t/T$, t is the period of rotation of the hoop, and T the half-life of the activity.

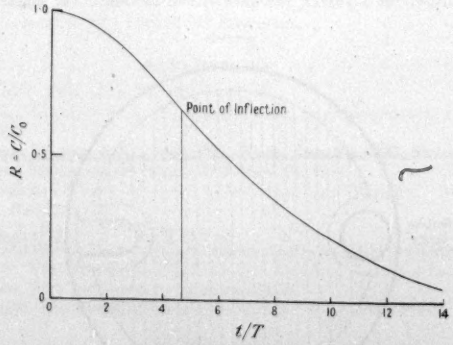

Figure 2. Variation of the counting rate with the period of rotation of the hoop.

The ratio $R = C/C_0$ is plotted against t/T in Figure 2, from which it will be seen that R differs little from 1 when $t = T$, and falls to half only when $t = 6{\cdot}3T$. Quite short-lived activities can therefore be investigated with relatively slow rotations of the hoop.

The ratio of the counting rates at any two periods of rotation will in principle enable T to be determined. In practice, one of the periods is made short enough to make R approach unity to high accuracy; then not only is the counting rate a maximum, but it is independent of the exact period of rotation. The slower period of rotation should of course be chosen so that R falls well below unity, and in a region where R varies most rapidly with t/T. More confidence will be felt if a number of different "long" periods are used and are found to lead to a statistically consistent value for T, but the weighting of the values obtained is a complicated matter. If, however, the chosen values of t/T are restricted to the practically straight portion about the point of inflection of the curve of Figure 2, corresponding values of R and t may be fitted to a straight line by the usual "least squares" method. Since the point of intersection of this line with the R axis can be calculated from equation (2), there is no loss of accuracy due to the restricted range of the experimental points; the most probable value of T and its statistical error can then be readily obtained.

§4. EXPERIMENTAL DETAILS

A period of rotation of 0·1 sec/rev. was used to measure the maximum counting rate C_0 due to the ^{6}He activity, which was then within 0·1% of the maximum value. The observed value of C_0 was 60 counts per minute, and the background due to the radiation from the source was 80 counts per minute.

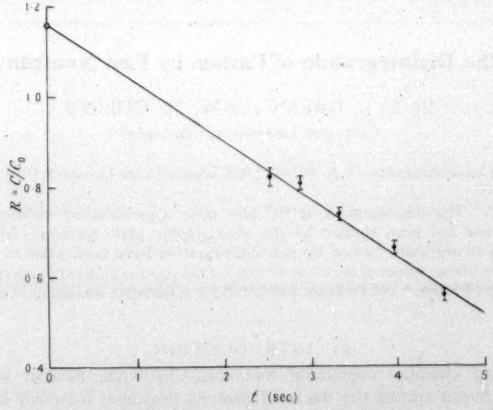

Figure 3. Experimental results with ^{6}He.

The ratio R was measured for five periods of rotation of the hoop between 2·5 and 4·6 sec/rev. The direction of rotation of the hoop was reversed for each measurement of R, to reduce any error due to lack of symmetry in the apparatus. The counting rate for each particular period of rotation was taken over a ten-minute interval. Ten measurements of R were made for each of the five values of t, giving fifty independent observations of R, from which an accuracy rather better than 2% was anticipated for the half-life.

The difference between the linear approximation and equation (2) over the range of t used was not greater than 0·25%; this error was allowed for in the estimated error of the final value for T. The simplification of the equation for the counting rate (eqn. (1)) does not introduce an error greater than 0·1%; this was checked by using estimated values of α and β, the effective angles subtended by the source and the counter respectively.

§ 5. CONCLUSION

The results obtained are shown graphically in Figure 3. The half-life of ^{6}He obtained by the "least squares" method from the experimental results is $T = 0{\cdot}823 \pm 0{\cdot}013$ sec.

The statistical accuracy of this result is considerably higher than that of previous determinations, which agree within their limits of error.

ACKNOWLEDGMENT

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REFERENCES

BJERGE, T., 1936, *Nature, Lond.*, **137**, 400.
CASSELS, J. M., and LATHAM, R., 1947, *Nature, Lond.*, **159**, 367.
NAHMIAS, M. E., and WALEN, R. J., 1937, *J. Phys. Radium*, **8**, 153.
SOMMERS, H. S., and SHERR, R., 1946, *Phys. Rev.*, **69**, 21.

The Disintegration of Carbon by Fast Neutrons

By L. L. GREEN and W. M. GIBSON
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Communicated by O. R. Frisch; MS. received 13th December 1948

ABSTRACT. The disintegration of ^{12}C into three α-particles by inelastic scattering of fast neutrons has been studied by the photographic plate method. Measurements made on 168 of the stars formed by this disintegration have been used to identify the reaction and to obtain values of the cross-section for the process at neutron energies between 10·8 MEV. and 14·5 MEV. No evidence was found for anisotropic scattering of the neutrons.

§ 1. INTRODUCTION

IN a cloud chamber containing methane, Chadwick, Feather and Davies (1934) found among the tracks of protons projected forwards by radium–beryllium neutrons a single small star consisting of three short tracks. They attributed this to the reaction $^{12}_{6}\mathrm{C} + {}^{1}_{0}\mathrm{n} \rightarrow 3\,{}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}$ in which a ^{12}C nucleus is split into three α-particles by a neutron which is itself inelastically scattered.

Nine similar three-pronged stars were found by Aoki (1938), who was using a cloud-chamber containing methane to investigate the neutrons produced in the bombardment of lithium by deuterons. He calculated the total energy for the three particles producing each star, assuming them to be α-particles, and obtained values which were consistent with the known maximum energy of the Li + D neutrons and the masses of the ^{12}C nucleus and the α-particle.

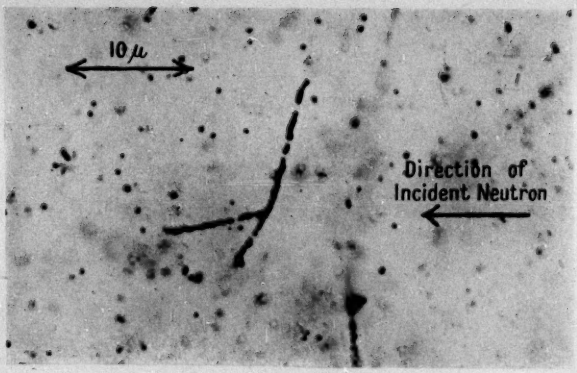

Figure 1. Photomicrograph of $^{12}C \rightarrow 3\alpha$ disintegration.

§ 2. DISINTEGRATIONS IN PHOTOGRAPHIC PLATES

We have observed a total of 168 stars, each consisting of three short tracks, while studying by the photographic plate method the spectrum of the neutrons emitted in the bombardment of a lithium target with deuterons of energy 0·93 MeV. (Green and Gibson 1949). A photomicrograph of one of these stars is shown in Figure 1. The fact that similar stars appeared in plates exposed to $^{11}B + D$ neutrons, but not in plates exposed to $D + D$, $N + D$ or $^{10}B + D$ neutrons, indicated that they were due to disintegrations of nuclei already present in the emulsion by neutrons of energy greater than 7 MeV. This fits with the Q-values of the only two possible reactions:

$$^{12}C + {}^{1}_{0}n \rightarrow 3\,{}^{4}_{2}He + {}^{1}_{0}n - 7{\cdot}34 \text{ MeV}. \quad \ldots\ldots(1)$$

$$^{14}_{7}N + {}^{1}_{0}n \rightarrow {}^{7}_{2}Li + 2\,{}^{4}_{2}He - 8{\cdot}8 \text{ MeV}. \quad \ldots\ldots(2)$$

It is known (Williams, Shepherd and Haxby 1937) that the lifetime of ^{5}He is too short for a nucleus formed during reaction (1) to make a track before breaking up into a neutron and an α-particle.

The ^{7}Li nucleus formed in reaction (2) would give a short track indistinguishable from that of an α-particle; but this reaction was in fact ruled out in the majority of cases by a detailed examination of the energies and the momenta of the particles.

§ 3. MEASUREMENTS AND CALCULATIONS

Detailed measurements were made on 66 of the 80 stars observed in plates exposed to the neutrons emerging from the lithium target in the direction of the incident deuterons, and on 38 of the 88 stars produced by neutrons emerging

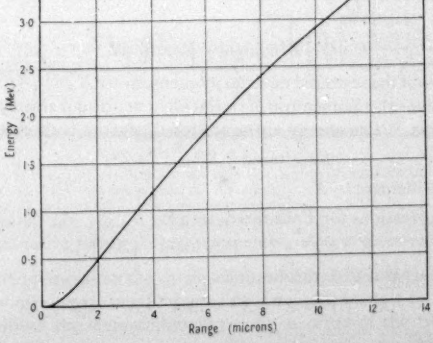

Figure 2. Range–energy relation for α-particles in Ilford C2 emulsion.

from the target at 120° to this direction. The length of each track and its direction relative to that of the incident neutrons in both vertical and horizontal planes were measured under a magnification of 1,800. The corresponding energy was obtained from an α-particle range–energy relation constructed from the data

of Holloway and Livingston (1938) for air and of Lattes, Fowler and Cuer (1947) for emulsion. The curves published by the latter workers are accurate only at energies above 2 MEV., and for lower energies it was necessary to use Holloway and Livingston's curve; the stopping-power at each energy was estimated from the values above 2 MEV., on the principles indicated by Rutherford, Chadwick and Ellis (1930). The range–energy relation used is shown in Figure 2.

From the directions and energies of the α-particles their total momentum was computed and taken to be equal to the (vector) difference of the momenta of the incident and scattered neutron. On the other hand the difference of the energies of the neutron was taken to be $E+Q$, where E is the total energy of the three α-particles and Q is 7·34 MEV., the energy required to split a ^{12}C nucleus into three α-particles (Mattauch 1942). In this way it was possible to obtain the energies of the incident and scattered neutrons and the direction of the scattered neutron relative to the known direction of the incident neutron. The method of calculation may be made clearer by an example.

The lengths (a) of the tracks, projected on a horizontal plane, were	9·7 μ,	4·0 μ,	4·75 μ
The dips (h), corrected for the shrinkage of the emulsion, were	−1 μ,	−2 μ,	0
The angles in a horizontal plane relative to the incident neutron direction were	1°,	−86°,	70°
The true lengths in space of the tracks ($L=\sqrt{(a^2+h^2)}$) were	10·2 μ,	4·47 μ,	4·75 μ
These gave energies (E_α) of	3·00,	1·30,	1·38 MEV.
	with a total of $E=5{\cdot}68$ MEV.		
The momenta in arbitrary units ($M=\sqrt{(AE)}$ for mass number A and energy E) were	3·46,	2·28,	2·35

These were resolved into three components, M_x parallel to the incident neutron direction, M_y perpendicular to this in the plane of the emulsion, and M_z vertical. The components of the total momentum were $X=+4{\cdot}32$, $Y=+0{\cdot}23$, $Z=-1{\cdot}36$.

The energy relation is

$$E+Q=E\,\text{incident}-E\,\text{scattered}. \qquad \ldots\ldots(3)$$

The components of the scattered neutron momentum are $X'=\{\sqrt{(E\,\text{incident})}-X\}$, $-Y$, and $-Z$, since the momentum of the incident neutron is simply $\sqrt{(E\,\text{incident})}$ in the X direction. The energy of the scattered neutron is therefore

$$E\,\text{scattered}=X'^2+Y^2+Z^2, \qquad \ldots\ldots(4)$$

where $X'=\sqrt{(E\,\text{incident})}-X$.

Equating the expressions for E scattered, we get

$$X'^2+Y^2+Z^2=E\,\text{incident}-(E+Q).$$

With $E\,\text{incident}=(X+X')^2$, this becomes

$$X'^2+Y^2+Z^2=X^2+2XX'+X'^2-(E+Q),$$

from which we get

$$X'=\frac{(E+Q+Y^2+Z^2)-X^2}{2X}. \qquad \ldots\ldots(5)$$

When X' is obtained, equation (4) gives E scattered and equation (3) then gives E incident. In the above example, the values of X, Y, Z, E and Q gave $X'=-0{\cdot}35$; from this, equations (3) and (4) gave 15·0 MEV. and 2·0 MEV. respectively for the energies of the incident and scattered neutrons. The

direction ϕ of the scattered neutron was found to be 104°, from the relation $\tan\phi = \{\sqrt{(Y^2+Z^2)}\}/X'$.

The number of cases in which the neutron was scattered forwards was compared with the number in which the scattering angle was more than 90° in centre-of-mass space. The ratio obtained was $1{\cdot}5 \pm 0{\cdot}5$, and this cannot be taken as evidence for a departure from isotropy.

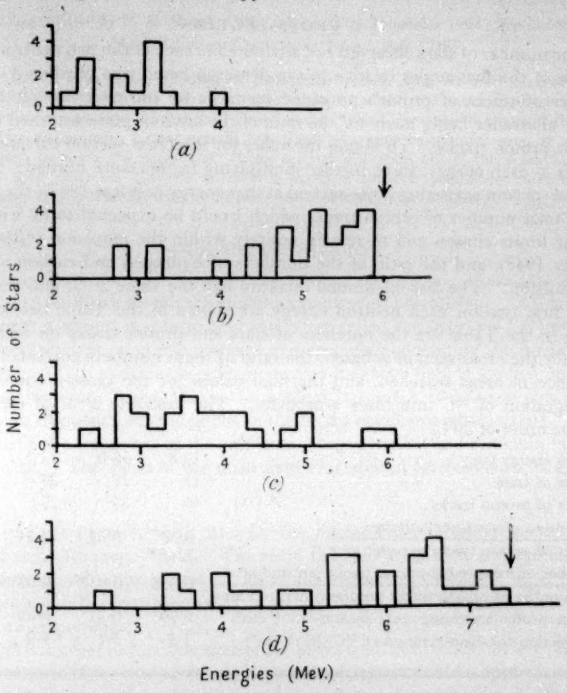

Figure 3. (*a*) refers to the stars attributed to the 10·8 MeV. group of neutrons in the 120° direction,
(*b*) to the 13·2 MeV. group in the 120° direction,
(*c*) to the 12 MeV. group in the forward direction, and
(*d*) to the 14·5 MeV. group in the forward direction.
The arrows indicate the maximum possible values of the energy.

It is difficult to measure the angle of dip accurately for such short tracks, and this led to some uncertainty in the incident neutron energy, but it was usually possible to attribute the stars definitely to one or other of the two high-energy groups of the Li + D neutron spectrum. The measurements were not sufficiently accurate to yield any definite information about the energy levels of intermediate nuclei (especially ^{8}Be and ^{5}He) which might be formed during the reaction. They were, however, accurate enough to show that if the alternative reaction (2) were assumed, no reasonable range–energy relation for ^{7}Li nuclei would make it possible to satisfy the laws of conservation of energy and momentum in more than about 10% of the cases.

In Figure 3 are shown, in histogram form, the distributions of the quantity E, the sum of the energies of the three α-particles, for the stars of each group. Also shown are the maximum possible values of E, obtained from the excess of the known maximum incident neutron energy at each angle over the Q-value of 7·34 MeV. which is given by the masses. The maximum observed values of E fall just below these limits, as expected.

§4. CROSS-SECTION

The number of stars observed and attributed to each of the two neutron groups at each of the two angles relative to the deuteron beam was compared with the number of tracks of protons projected forwards by the neutrons of the same group, allowance being made for the ratio of the areas of plate searched for stars and for proton tracks. To obtain the value for the cross-section for production of stars at each energy, three further multiplying factors were needed: the total neutron–proton scattering cross-section at that energy (Sleator 1947), the fraction of the total number of proton tracks which would be expected to lie within the angular limits chosen and to remain entirely within the emulsion (Gibson and Livesey 1948), and the ratio of the numbers of hydrogen and carbon atoms in the emulsion. The last-mentioned quantity had the value 2·03, and the values of the first two for each neutron energy are shown in the Table below. Also shown in the Table are the numbers of stars and proton tracks on which each value for the cross-section is based, the ratio of these numbers corrected for the difference in areas searched, and the final values for the cross-section for the disintegration of ^{12}C into three α-particles. The probable error of each value is of the order of 20%.

Neutron energy (MeV.)	10·8	12·0	13·2	14·5
Number of stars	13	27	24	23
Number of proton tracks	46	37	25	12
$\frac{\text{Number of stars per unit area}}{\text{Number of proton tracks per unit area}}$	0·37	0·65	2·34	5·25
$\frac{\text{Number of measurable proton tracks per unit area}}{\text{Total number of proton tracks produced per unit area}}$	0·038	0·032	0·0275	0·023
Neutron–proton scattering cross-section (10^{-24} cm^2)	0·79	0·73	0·69	0·64
Cross-section for disintegration of ^{12}C (10^{-26} cm^2)	2·3	3·1	9·0	15·7

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REFERENCES

Aoki, H., 1938, *Proc. Mat.-Phys. Soc., Japan*, **20,** 755.
Chadwick, J., Feather, N., and Davies, W. T., 1934, *Proc. Camb. Phil. Soc.*, **30,** 357.
Gibson, W. M., and Livesey, D. L., 1948, *Proc. Phys. Soc.*, **60,** 523.
Green, L. L., and Gibson, W. M., 1949, to be published.
Holloway, M. G., and Livingston, M. S., 1938, *Phys. Rev.*, **54,** 18.
Lattes, C. M. G., Fowler, P. H., and Cuer, P., 1947, *Proc. Phys. Soc.*, **58,** 883.
Mattauch, J., 1942, *Kernphysikalische Tabellen* (Berlin: Springer).
Rutherford, E., Chadwick, J., and Ellis, C. D., 1930, *Radiations from Radioactive Substances* (Cambridge: University Press).
Sleator, W., 1947, *Phys. Rev.*, **72,** 207.
Williams, J. H., Shepherd, W. G., and Haxby, R. O., 1937, *Phys. Rev.*, **52,** 390.

A Direct Method of Measuring Nuclear Spin-Lattice Relaxation Times

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Communicated by F. E. Simon; MS. received 27th November 1948; *read at the Physical Society Conference, Oxford, July* 1948.

ABSTRACT. This direct method of measuring nuclear spin–lattice relaxation times is based on Bloch's nuclear induction experiment in which the material investigated is subjected to a magnetic field that is varied several times per second through the value for nuclear resonance. Under certain conditions a simple calculation may be made of the relation between the magnitude of the nuclear induction effect, the relaxation time, and the time intervals between successive passages through resonance. Experimental observation of this relation then determines the relaxation time directly.

§ 1. INTRODUCTION

THIS direct method of measuring nuclear spin–lattice relaxation times is based on Bloch's nuclear induction experiment (Bloch 1946). In this experiment a strong magnetic field (hereafter called the main field H) produces a resultant nuclear magnetic moment in a specimen containing the nuclei under investigation. In addition, a radio-frequency magnetic field is applied perpendicular to the main field. When the applied frequency is close to the Larmor precession frequency for the nuclei in the field a resonance effect occurs.

Usually the frequency of the oscillator is kept constant and the main magnetic field varied. The value of the main field H_r required for resonance is given by

$$|\gamma| H_r = 2\pi\nu,$$

where γ is the gyromagnetic ratio for the nuclei and ν is the frequency of the applied radio-frequency field. The main field is varied over a range including the resonant value by means of an audio-frequency "sweep" field (of, say, 50 cycles/second) superimposed on a steady field. The resonant value is thus passed twice per cycle.

Bloch's nuclear induction experiment differs from most other nuclear resonance experiments in that a comparatively large radio-frequency field is applied, sufficient in fact to cause a considerable change in the nuclear magnetization in one passage through the resonant region, though still small compared with the main field. It can in fact be shown that, if the radio-frequency field is large enough, the nuclear magnetization is actually reversed, or turned over completely, in varying the main field through the resonant value. For this to be true it is necessary to have the following condition satisfied (Bloch's adiabatic condition) :

$$dH/dt \ll |\gamma| H_1^2, \qquad \ldots\ldots (1)$$

where $2H_1$ is the amplitude of the radio-frequency field and dH/dt the rate of change of the main field. In this case, when the main field is near the resonant value, the nuclear magnetization precesses about the main field with the applied frequency and there exists a rotating transverse component of the magnetic moment M_t given by

$$M_t = M(1+\delta^2)^{-\frac{1}{2}} \quad \text{where} \quad \delta = (H - H_r)/H_1, \qquad \ldots\ldots (2)$$

where M is the magnitude of the nuclear moment which remains constant during the passage through resonance if relaxation effects can be neglected. (M is the vector sum of the instantaneous values of the individual nuclear moments, and is in general not equal to the equilibrium value M_0). The rotating transverse magnetization induces a radio-frequency E.M.F. in a secondary coil. This E.M.F.,

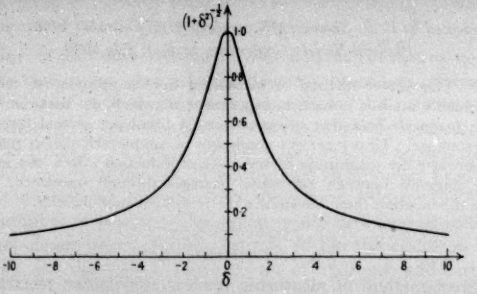

Figure 1. Shape of nuclear induction signal in the case of rapid passage.

after detection and amplification, is put on the Y-plates of a cathode-ray oscillograph. To the X-plates is applied a voltage which is proportional to, and in phase with, the alternating component of the main field. The shape of the signal obtained when the main field passes through its resonant value is shown in Figure 1, which is a plot of the rotating transverse magnetization against the main field.

§ 2. RELAXATION EFFECTS

The relaxation times affecting nuclear magnetism are that for spin–spin equilibrium (Bloch's transverse relaxation time) and that for spin–lattice equilibrium (Bloch's longitudinal relaxation time). In the cases we shall consider, the relaxation times are long compared with the time taken to pass through the resonant region (i.e. the case of rapid passage (Bloch 1946)). In this case the shape of the signals is not altered appreciably by relaxation effects, and if the time spent in resonance is small compared with the period of the audio-frequency alternating field, or sweep, the relaxation effects can be considered as taking place only outside resonance. This is a great simplification and means that only the longitudinal or spin–lattice relaxation time has to be considered, for it will be seen that transverse relaxation can only be effective in the resonant region, for it is only then that there exists a transverse magnetization. In what follows, the term " relaxation time " will refer to the ordinary longitudinal or spin–lattice relaxation time. We may distinguish between two cases: (i) relaxation time long compared with the period of sweep; (ii) relaxation time of the same order as the period of sweep.

In the first case the magnitude and sign of the signals when equilibrium has been attained depends on the relative times spent above and below resonance: the nuclear magnetization will be in the direction of the field during whichever of the two parts of the cycle is greater. The dependence of the size of the signal on its position on the trace of the cathode-ray oscillograph (i.e. on the position of the resonant field with respect to the range of variation of the main field) is shown in Figure 2. It will be seen that the forward and backward traces are

practically the same, since the magnetization has not time to alter appreciably during the time between successive passages through resonance. If the relaxation time is longer than about 1 second, the time taken for the system to adjust itself to new conditions, e.g. to a change in the mean value of the main field, can be seen visually on the cathode-ray tube screen, and the relaxation time obtained easily with the aid of a stop-watch.

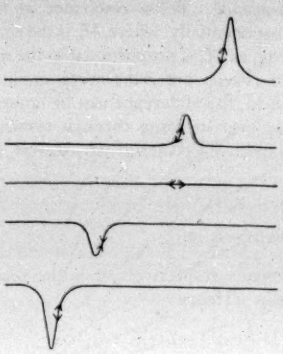

Figure 2. Variation of size of signal with position on trace. Long relaxation time.

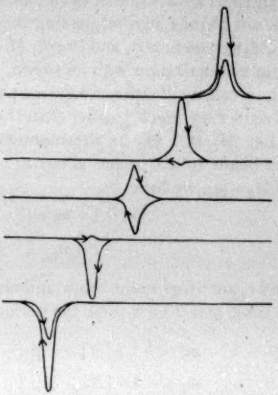

Figure 3. Variation of size of signal with position on trace. Relaxation time equal to the period of sweep.

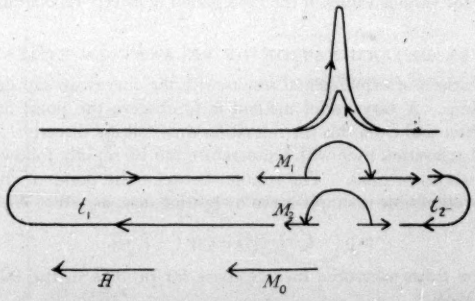

Figure 4. Diagram showing behaviour of the nuclear magnetization during one cycle of the sweep. M_0 is equilibrium value of the nuclear magnetization in a steady field.

In the second case, that is with the relaxation time comparable with the period of sweep, there is time for the magnetization to change during the intervals below and above resonance. The variation of the signal with its position on the trace may be as in Figure 3, which illustrates the case of the relaxation time and period of sweep equal.

In Figure 4 the behaviour of the nuclear magnetic moment during a cycle of the audio-frequency sweep field is illustrated diagrammatically. On the

assumption that the nuclear magnetization tends always to approach its equilibrium value exponentially, a calculation may be made of the signals to be expected. It is evident that the size of the signal in a passage through resonance is directly proportional to the magnetic moment existing just before the passage. For convenience, the nuclear magnetization M is considered positive if it is parallel to the field below resonance and antiparallel above resonance; thus the sign of the signal is always that of the moment turned over, and the nuclear magnetization does not change sign on passing through resonance. Below resonance we have $M \rightarrow M_0$ exponentially, and above, $M \rightarrow -M_0$ exponentially, where M_0 is the equilibrium magnetization with no sweep. (Strictly, as M_0 is proportional to the main field, its value will differ at different points of the sweep, but as the sweep amplitude is always very much smaller than the main field, this difference can be ignored.)

Let M_1 and M_2 be the moments turned over in going through resonance from below to above and above to below respectively; then if $M_1/M_0 = m_1$ and $M_2/M_0 = m_2$, we have

$$\left.\begin{aligned} 1-m_1 &= (1-m_2)\exp(-t_1/\tau) \\ -1-m_2 &= (-1-m_1)\exp(-t_2/\tau). \end{aligned}\right\} \quad \ldots\ldots (3)$$

t_1 and t_2 are times spent below and above resonance respectively, τ is the relaxation time, and $T\,(=t_1+t_2)$, the period of sweep. Hence

$$\left.\begin{aligned} m_1 &= -1+[2(1-\exp\{-t_1/\tau\})]/[1-\exp\{-T/\tau\}], \\ m_2 &= 1-[2(1-\exp\{-t_2/\tau\})]/[1-\exp\{-T/\tau\}]. \end{aligned}\right\} \quad \ldots\ldots (4)$$

In Figure 5 m_1/ and m_2 are plotted for various positions on the trace assuming a sinusoidal sweep. The graph shows how the signal size varies with the position on the trace for various values of the ratio period of sweep–relaxation time.

§ 3. DETERMINATION OF RELAXATION TIMES

By comparison of experimental results with the curves we can determine the relaxation time. A very useful method is to observe the point at which one signal vanishes, and from this the relaxation time follows directly. In this way variations of relaxation time with temperature can be rapidly followed in a substance as it warms or cools. The relation between the point of disappearance and the relaxation time is simply given by putting, say, $m_2 = 0$. We get

$$\exp(-t_2/\tau) = \tfrac{1}{2}[1+\exp(-T/\tau)]. \quad \ldots\ldots (5)$$

Relaxation times measured include those for protons in the following substances at 8 Mc/s.:

Paraffin wax at room temperature, 0·02 sec.
Glycerine over the range $-20°$ to $+50°$ c., 0·2 sec. (54° c.), 0·005 sec. ($-12°$ c.)
Liquid hydrogen at boiling point, 0·2–0·5 sec.
Distilled water at room temperature, 2·7 sec. (stop-watch method).

The results for glycerine agree fairly well with those obtained by Bloembergen, Purcell and Pound (1947) by the saturation method in nuclear resonance absorption. Below $-20°$ c. the method failed, for the signals became smaller and the results were not independent of the amplitude of sweep. Possibly the reason

for this was that the transverse relaxation time was becoming comparable with the time spent in resonance. The results for glycerine are shown in Figure 6,

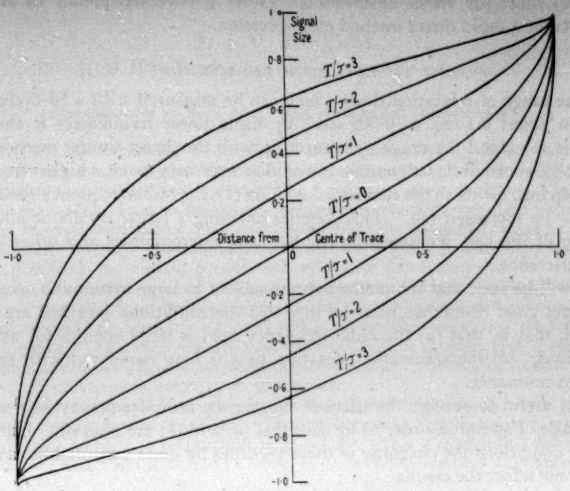

Figure 5. Nuclear induction experiment.
Dependence of signal size on position of signal on trace.
T = Period of modulating field. τ = Relaxation time of specimen.

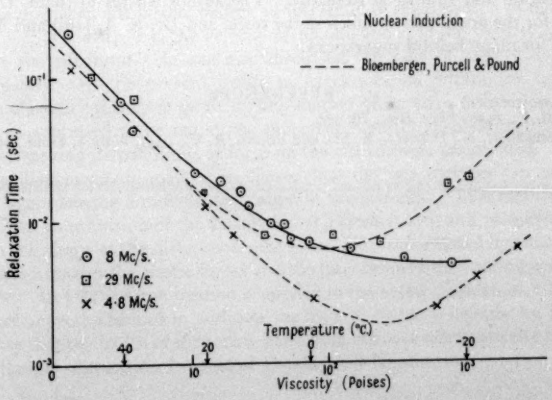

Figure 6. Graph showing variation of the nuclear spin–lattice relaxation time for protons in glycerine. Relaxation time is plotted against viscosity.

in which relaxation time is plotted against viscosity, following Bloembergen, Purcell and Pound. Their results are given for comparison.

The value given for liquid hydrogen is only approximate as the relaxation time is too long for measurement by the method given, with a 50-cycle sweep, but too short for visual observation. Either a lower frequency of sweep is required or a rapid direct method of observation.

§4. CONCLUSION

The range of relaxation times which can be measured with a 50-cycle sweep is from about 0·1 sec. to 0·005 sec. By using lower frequencies it should be possible to extend the range to connect up with the direct timing method. To determine shorter relaxation times it would be necessary to use a higher frequency of sweep but, owing to the adiabatic condition (1), the radio-frequency field would have to be increased too. This, besides entailing a larger oscillator and more heating of the coil, increases the trouble from microphonics and other sources of interference.

It will be seen that the method is not subject to large systematic errors, but to ensure good results we must be sure that the conditions specified are in fact obeyed, that is, that (*a*) the radio-frequency field is large enough for adiabatic conditions; (*b*) the transverse relaxation time is long compared with the time spent in resonance.

It is useful to observe the effect of varying the radio-frequency field and the amplitude of sweep in order to be sure that (*a*) and (*b*) are obeyed. Under the correct conditions the changing of these variables by quite considerable amounts should not affect the results.

ACKNOWLEDGMENTS

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REFERENCES

Bloch, F., 1946, *Phys. Rev.*, **70**, 460.
Bloembergen, N., Purcell, E. M., and Pound, R. V., 1947, *Nature, Lond.*, **160**, 475.

Uniformly Moving Dislocations

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ABSTRACT. An expression is derived for the displacements in an isotropic elastic medium which contains an edge dislocation moving with uniform velocity c. When $c=0$ the solution reduces to that given by Burgers for a stationary edge dislocation. The energy density in the medium becomes infinite as c approaches c_2, the velocity of shear waves in the medium; this velocity therefore sets a limit beyond which the dislocation cannot be accelerated by applied stresses. The atomic structure of the medium is next partly taken into account, following the method already used by Peierls and Nabarro for the stationary dislocation. The solution found in this way differs from the one in which the atomic structure is neglected only within a region of width ζ which extends not more than a few atomic distances from the centre. ζ varies with c and vanishes when $c=c_r$, the velocity of Rayleigh waves. It becomes negative when $c_r<c<c_2$. Thus c_r rather than c_2 appears to be the limiting velocity when the atomic nature of the medium is taken into account. Since $c_r\simeq 0{\cdot}9c_2$ the difference is not of much importance.

The same method applied to a screw dislocation gives, in the purely elastic case, the expression already derived by Frank. The corresponding Peierls–Nabarro calculation shows that the width ζ is proportional to $(1-c^2/c_2^2)^{\frac{1}{2}}$. This "relativistic" behaviour is analogous to Frenkel and Kontorowa's results for their one-dimensional dislocation model.

§1. INTRODUCTION

FRANK (1949) has discussed the motion of screw dislocations: he shows that the displacement system suffers a contraction in the direction of motion as $(1-c^2/c_2^2)^{\frac{1}{2}}$ where c is the velocity of motion and c_2 is the velocity of shear waves in the medium. He has also discussed the case of edge dislocations approximately. In the present paper an exact solution within the limits of isotropic elasticity theory is given for the motion of an edge dislocation. The material is supposed to be cut at the slip plane, and the displacements due to uniformly moving distributions of force on the cut surfaces are derived. These travelling displacements may be so chosen that the two surfaces can be stuck together again leaving a moving dislocation in the material. This method lends itself (§3) to an extension of the calculation of Peierls (1940) and Nabarro (1947) in which the shape of the dislocation near its centre is determined by considering the balance between the elastic forces and the interatomic forces acting across the slip plane. In §4 the same method is applied to the screw dislocation. Frank's results are reproduced and in addition the Peierls–Nabarro calculation for this case shows that the width of the screw dislocation behaves relativistically just as it does in the one-dimensional model of Frenkel and Kontorowa.

§2. MOVING EDGE DISLOCATION

Imagine the infinite unstressed elastic body of Figure 1 to be cut in two along the plane $y=0$ and the two halves slightly separated. Let tangential forces $p(x)$ per unit area be applied to the surface AB, independent of z and everywhere parallel to the x-axis, and producing horizontal and vertical displacements u, v

at the surface. Now let exactly equal opposite forces $-p(x)$ be applied at corresponding points on CD producing displacements $\bar{u}=-u$, $\bar{v}=v$. The two halves can be welded together and the forces p removed, leaving the body in a "self-stressed" state. A knowledge of u along the x-axis is sufficient to determine u and v throughout the solid.

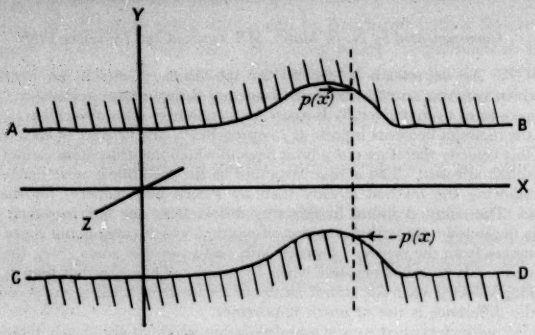


Figure 1.

The particular choice

$$u=-\bar{u}=0, \quad x>0; \qquad u=-\bar{u}=\lambda_0/2, \quad x<0 \qquad \ldots\ldots(1)$$

for the surface displacements gives the edge-type dislocation of Burgers (1939) for which

$$\left.\begin{aligned} u&=\frac{\lambda_0}{2\pi}\tan^{-1}\frac{y}{x}+\frac{\lambda_0}{2\pi}\frac{\lambda+\mu}{\lambda+2\mu}\frac{xy}{x^2+y^2},\\ v&=-\frac{\lambda_0}{2\pi}\frac{\mu}{\lambda+2\mu}\ln(x^2+y^2)^{\frac{1}{2}}+\frac{\lambda_0}{2\pi}\frac{\lambda+\mu}{\lambda+2\mu}\frac{y^2}{x^2+y^2}, \end{aligned}\right\} \qquad \ldots\ldots(2)$$

where λ and μ are the Lamé constants and λ_0 is the "strength" of the dislocation. u increases by λ_0 on describing a circuit around the origin.

Suppose now that the distributions of force p and $-p$ are moving uniformly along the x-axis with velocity c, so that they depend on x and t only through $x'=x-ct$. The corresponding displacements will be of the form $u(x',y)$, $v(x',y)$; their connection with each other and with $p(x')$ can be found by an extension of the analysis used in discussing Rayleigh surface waves (Love 1944).

Consider the displacements

$$u_1=Ae^{-\gamma s y}\sin sx', \qquad v_1=A\gamma e^{-\gamma s y}\cos sx',$$

$$u_2=B\beta e^{-\beta s y}\sin sx', \qquad v_2=Be^{-\beta s y}\cos sx',$$

where

$$\beta=(1-c^2/c_2^2)^{\frac{1}{2}}, \qquad \gamma=(1-c^2/c_1^2)^{\frac{1}{2}}, \qquad x'=x-ct,$$

and $c_1=\{(\lambda+2\mu)/\rho\}^{\frac{1}{2}}$, $c_2=(\mu/\rho)^{\frac{1}{2}}$ are the velocities of longitudinal and shear waves in the medium, whose density is ρ. u_1 and v_1 correspond to an irrotational disturbance and satisfy

$$(c_1^2\nabla^2-\partial^2/\partial t^2)(u_1, v_1)=0, \qquad \partial u_1/\partial y-\partial v_1/\partial x=0, \qquad \ldots\ldots(3)$$

while u_2 and v_2 correspond to a distortional disturbance and satisfy

$$(c_2^2\nabla^2-\partial^2/\partial t^2)(u_2, v_2)=0, \qquad \partial u_2/\partial x+\partial v_2/\partial y=0,$$

and consequently the combination $u=u_1+u_2$, $v=v_1+v_2$ satisfies the equations governing the motion of an elastic solid, namely

$$(\lambda+\mu)\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\mu\nabla^2 u-\rho\frac{\partial^2 u}{\partial t^2}=0$$

and a similar equation for v. The condition that the normal stress

$$p_{yy}=\lambda\partial u/\partial x+(\lambda+2\mu)\partial v/\partial y$$

should vanish at the surface $y=0$ is easily found to be $B/A=-\alpha^2/\beta$, where $\alpha=(1-c^2/2c_2^2)^{\frac{1}{2}}$. Corresponding u and v are then

$$u=(2c_2^2/c^2)(e^{-\gamma sy}-\alpha^2 e^{-\beta sy})\sin sx', \qquad \ldots\ldots(4)$$

$$v=(2c_2^2/c^2)(\gamma e^{-\gamma sy}-\alpha^2 e^{-\beta sy}/\beta)\cos sx', \qquad \ldots\ldots(5)$$

the amplitude of u being chosen to be unity for $y=0$. The shear stress at the surface must be equal to the applied force $p(x')$; it is easily found to be

$$p_{xy}(x', 0)=p(x')=D(c)s\sin sx', \qquad \ldots\ldots(6)$$

where $D(c)=-2\mu(2c_2^2/c^2)(\gamma-\alpha^4/\beta)$. Solving $D(c)=0$ gives the velocity of Rayleigh waves.

If a more general travelling surface wave is built up in the form

$$u(x', 0)=\int_0^\infty f(s)\sin sx'\,ds,$$

then

$$u(x', y)=(2c_2^2/c^2)\int_0^\infty (e^{-\gamma sy}-\alpha^2 e^{-\beta sy})f(s)\sin sx'\,ds, \qquad \ldots\ldots(7)$$

$$v(x', y)=(2c_2^2/c^2)\int_0^\infty (\gamma e^{-\gamma sy}-\alpha^2 e^{-\beta sy}/\beta)f(s)\cos sx'\,ds, \qquad \ldots\ldots(8)$$

$$p(x')/D(c)=\int_0^\infty f(s)s\sin sx'\,ds. \qquad \ldots\ldots(9)$$

If u, v are split up into u_1, v_1 and u_2, v_2, the suffix 1 referring to the parts containing γ and 2 to those containing β, then γu_1, v_1 are Hilbert transforms (Titchmarsh 1937) and so are u_2, βv_2. A more useful relation between them is the following. Equations (3), (4) and (5) give

$$\left(\frac{\partial^2}{\partial x'^2}+\frac{\partial^2}{\partial(\gamma y)^2}\right)(\gamma u_1, v_1)=0, \qquad -\frac{\partial(\gamma u_1)}{\partial x'}=\frac{\partial v_1}{\partial(\gamma y)}, \qquad \frac{\partial(\gamma u_1)}{\partial(\gamma y)}=\frac{\partial v_1}{\partial x'},$$

and this will continue to hold for the more general u_1, v_1. By the usual theory of conjugate functions it follows that if

$$\gamma u_1=\mathrm{Im}\,P(x+i\gamma y) \quad \text{then} \quad v_1=\mathrm{Re}\,P(x+i\gamma y), \qquad \ldots\ldots(10)$$

and similarly if

$$u_2=\mathrm{Im}\,Q(x+i\beta y) \quad \text{then} \quad \beta v_2=\mathrm{Re}\,Q(x+i\beta y), \qquad \ldots\ldots(11)$$

where P and Q are analytic functions. (The relation between Fourier transforms, Hilbert transforms and the complex variable are fully discussed by Titchmarsh).

Suppose that at the surfaces AB and CD u and $\bar{u}$ are given by (1) with x replaced by x'. This may be written

$$u = -\bar{u} = \frac{\lambda_0}{4} - \frac{\lambda_0}{4}\frac{2}{\pi}\int_0^\infty \frac{\sin sx'}{s}\,ds.$$

The first term clearly represents a constant displacement for all x', y. Thus by (7) for $y>0$

$$u(x',y) = \frac{\lambda_0}{4} - \frac{\lambda_0}{2\pi}\left(\frac{2c_2^2}{c^2}\right)\int_0^\infty (e^{-\gamma s y} - \alpha^2 e^{-\beta s y})\frac{\sin sx'}{s}\,ds$$

$$= \frac{\lambda_0}{2\pi}\frac{2c_2^2}{c^2}\left[\tan^{-1}\frac{\gamma y}{x'} - \alpha^2\tan^{-1}\frac{\beta y}{x'}\right]. \qquad \ldots\ldots(12)$$

Since

$$\ln(x+iy) = \ln(x^2+y^2)^{\frac{1}{2}} + i\tan^{-1}(y/x),$$

it follows from (10) and (11) that

$$v(x',y) = \frac{\lambda_0}{2\pi}\frac{2c_2^2}{c^2}\left[\gamma\ln(x'^2+\gamma^2y^2)^{\frac{1}{2}} - \frac{\alpha^2}{\beta}\ln(x'^2+\beta^2y^2)^{\frac{1}{2}}\right]. \qquad \ldots\ldots(13)$$

There is a corresponding expression for (u, v) when $y<0$ derived from $\bar{u}$. Except at the singularity at $x=0$ there is no relative motion between the planes AB and CD and they can be imagined to be welded together. It is easy to see that (12) and (13) give the displacements for $y<0$ as well as for $y>0$.

On passing to the limit $c=0$, (12) and (13) become (2). Independently of the method of derivation the displacements (12) and (13) have the following properties: (i) they satisfy the wave equation; (ii) u increases by λ_0 on describing any circuit round the point $x'=x-ct=0$; (iii) they reduce to the expressions for a stationary edge dislocation when $c=0$.

The irrotational and distortional terms behave relativistically with respective limiting velocities c_1 and c_2, but the proportion between them also alters with velocity. On account of the factor $1/\beta$ in the second term of v the energy becomes infinite as c approaches c_2 even if the singularities at $x'=0$ and infinity are excluded in the usual way. If solutions exist for $c>c_2$ it would be difficult to connect them continuously with the solution for $c<c_2$ and so identify them as moving edge dislocations; c_2 is thus the limiting velocity.

If (1) is replaced by

$$u = -\bar{u} = \left\{\frac{\lambda_0}{4}\left(1+\frac{2}{\pi}\tan^{-1}\frac{x'}{\zeta}\right), \qquad \ldots\ldots(14)\right.$$

to which it reduces when $\zeta\to 0$, a corresponding calculation shows that u and v in the body of the material are given by (12) and (13) on replacing y by $y\pm\zeta$, the sign of ζ to be chosen to be the same as that of y. Formally this "spread" dislocation is equivalent to a distribution of elementary dislocation of the type (12), (13), the total strength between x' and $x'+dx'$ being

$$d\lambda_1 = \frac{\lambda_0}{\pi}\frac{\zeta}{x'^2+\zeta^2}dx'. \qquad \ldots\ldots(15)$$

The change of u round a circuit depends on the size and position ot the circuit but approaches λ_0 for a large circuit surrounding $x'=0$.

At first sight it is surprising that the method used should lead to a moving form of the Burgers expression (2) rather than to a moving form of the original Taylor expression

$$u=\frac{\lambda_0}{2\pi}\tan^{-1}\frac{y}{x}, \qquad v=\frac{\lambda_0}{2\pi}\ln(x^2+y^2)^{\frac{1}{2}}, \qquad \ldots\ldots(16)$$

or something similar. This behaviour can be traced to the condition $B/A=-\alpha^2/\beta$ imposed to make p_{yy} vanish over the plane $y=0$. The actual condition used was that the Fourier transform of $p_{yy}(x',0)$ should vanish for all x'. Strictly this implies (Doetsch 1937) that $p_{yy}(x',0)$ vanishes "almost everywhere" and that any singularities are of such a feeble kind that $\int_0^X p_{yy}(x',0)\,dx'=0$ for any X. A singularity of the type of a Dirac δ-function is thus excluded. The last equation implies that $\int_{-\infty}^{\infty} p_{yy}(x',0)\,dx=0$ which in turn means that

$$\int_{-\infty}^{\infty} p_{yy}(x',y)\,dx=0, \qquad \ldots\ldots(17)$$

as can be seen by considering the equilibrium of a slab bounded by the planes $y=0$ and $y=$ const., or analytically by the method of Love (1944, §117). As is well-known (16) does not satisfy (17), which accounts for its non-appearance in the previous analysis.

The whole argument could in fact be repeated without requiring that $p_{yy}(x',0)=0$ everywhere, and the two halves of the material could be welded together as before and would be in equilibrium provided suitable external forces could be applied to the junction plane. Among other solutions (12), or the corresponding form with y replaced by $y\pm\zeta$, would reappear but with the coefficients of the inverse tangents arbitrary. The first or second terms of (12) and (13) taken separately do in fact satisfy the wave equation and give rise to the two following types of dislocations which could exist with suitable forces applied to the plane $y=0$:

$$u=\frac{\lambda_0}{2\pi}\tan^{-1}\frac{\gamma(y\pm\zeta)}{x'}, \qquad v=\frac{\lambda_0}{2\pi}\ln\{x'^2+\gamma^2(y\pm\zeta)^2\}^{\frac{1}{2}} \qquad \ldots\ldots(18)$$

and

$$u=\frac{\lambda_0}{2\pi}\tan^{-1}\frac{\beta(y\pm\zeta)}{x'}, \qquad v=\frac{\lambda_0}{2\pi}\frac{1}{\beta}\ln\{x'^2+\beta^2(y\pm\zeta)^2\}^{\frac{1}{2}}. \qquad \ldots\ldots(19)$$

Their limiting velocities are c_1 and c_2 respectively. For (18)

$$p_{yy}=\pm\frac{\lambda_0}{\pi}\mu\gamma\alpha^2\frac{\zeta}{x'^2+\zeta^2}$$

on the surfaces AB and BC (Figure 1) respectively. Thus to support it the dislocation requires a total external force

$$F_1=2\int_{-\infty}^{\infty} p_{yy}\,dx=2\lambda_0\mu\alpha^2$$

parallel to the y-axis distributed over a distance of the order ζ along the x-axis. When $\zeta=0$ the force is concentrated at the origin. For a velocity $c=\sqrt{2}c_2$, F_1 vanishes and the dislocation could exist without external forces provided $\sqrt{2}c_2$ is less than the limiting velocity c_1: the condition for this is $\lambda>0$, which is satisfied

in normal physical cases. This agrees with (12) and (13) on putting $c=\sqrt{2}c_2$. This isolated solution cannot be reached from the solutions for $c<c_2$. On the other hand (19) requires a total force $F_2=2\lambda_0\mu\beta$ which does not vanish below the limiting velocity. At any velocity less than c_2 a linear combination of (18) and (19) can exist provided the proportion between them is such that the external forces required to support each separately are equal and opposite. This is equivalent to the condition $B/A=-\alpha^2/\beta$ and leads back to (12) and (13).

When $c=0$ and $\zeta=0$ both (18) and (19) reduce to (16). The Taylor expression is thus both irrotational and dilatationless and requires for its maintenance a distribution of force along the z-axis of linear density $2\lambda_0\mu$ directed parallel to the y-axis. If it is to be in equilibrium in the slab defined by the planes $y=\pm Y$ there must be a normal pressure of total amount $\lambda_0\mu$ suitably distributed over the surface Y and a corresponding normal tension of total amount $\lambda_0\mu$ distributed over the surface $-Y$.

§ 3. THE WIDTH OF A MOVING EDGE DISLOCATION

By considering the surfaces AB and CD of Figure 1 not as welded together but as kept in place by interatomic forces across the slip-plane, Peierls (1940) and Nabarro (1947) were led to an expression of the form (14) (with x replacing x') for a stationary edge dislocation with a width $\zeta=\lambda_0/2(1-\sigma)$ depending on σ, Poisson's ratio. Briefly their argument was the following. Starting from the relation

$$u=\sin sx, \qquad p_{xy}=-\{\mu/(1-\sigma)\}s\sin sx, \qquad \ldots\ldots(20)$$

connecting a sinusoidal displacement of the surface AB with the tangential force necessary to produce it, and from an assumed law

$$p_{xy}=-(\mu/2\pi)\sin 2\pi(u-\bar{u})/\lambda_0, \qquad \ldots\ldots(21)$$

giving the atomic forces arising from the relative horizontal displacement of AB and CD at any point, they reach an integral equation for u which has (18) as a solution. The atomic nature of the medium only enters the problem through the periodicity of (21) and so the origin of x in (14) is arbitrary.

In so far as (21) is a good approximation in the stationary case it will also be nearly correct if du/dt is small compared with the natural velocity of vibration of the atoms, i.e. if the velocity of the dislocation is small compared with c_2. Even if c is of this order du/dt will only approach c near the centre of the dislocation where (21) is least accurate even in the static case. A repetition of the Peierls–Nabarro calculation for the moving dislocation may therefore have some significance. According to (4) and (6)

$$u=\sin sx', \qquad p_{xy}=D(c)s\sin sx' \qquad \ldots\ldots(22)$$

replaces (20), to which it reduces when $c=0$. This leads to (14) (with x replaced by x') where now

$$\zeta=\delta\lambda_0/2(1-\sigma), \qquad \delta=D(c)/D(0).$$

In Figure 2, δ is shown for $\sigma=\frac{1}{4}$, compared with the simple relativistic contraction factor β occurring in the case of the screw dislocation (§ 4). δ vanishes when $c=c_r$, the Rayleigh wave velocity.

According to the present model the isolated solution of § 2 with $c=\sqrt{2}c_2$ would have a complex ζ unless $\lambda=0$. In that case $\zeta=0$ and $\gamma=\alpha=0$ so that it

would exist in the completely contracted form $v=0$, $u=0$, $x'>0$; $u=\lambda_0/2$, $x'<0$, $y>0$; $u=-\lambda_0/2$, $x'<0$, $y<0$.

It is hardly worth while discussing what happens when $c_r<c<c_2$ since the approximations made will be very poor in that region. The statement at the end of § 2 should be qualified by saying that the limiting velocity is of the order of $0{\cdot}9c_2$.*

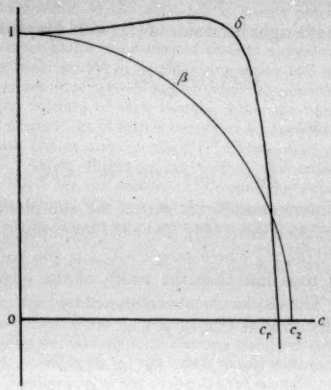


Figure 2. β and δ for $\sigma=\frac{1}{3}$ as functions of c.

Apart from any considerations of interatomic forces the dislocation of this section when ζ is any function of c which vanishes for $c=0$ could be considered as the moving form of (1). However, if it is required that the change of u shall be the same for any circuit surrounding $x'=0$ the form (12), (13) is unique.

§ 4. MOVING SCREW DISLOCATION

The screw dislocation already discussed by Frank may be treated in the same way, or more briefly by the following equivalent method.

If $\phi(x,y)$ satisfies $(\partial^2\phi/\partial x^2)+(\partial^2\phi/\partial y^2)=0$ then (Jeffreys and Jeffreys 1946, Titchmarsh 1937)

$$\phi(x,y)=\frac{1}{\pi}\int_{-\infty}^{\infty}d\xi\int_0^{\infty}\phi(\xi,0)e^{-sy}\cos s(x-\xi)\,ds.$$

A purely distortional elastic disturbance in which the displacement w is everywhere in the z-direction must satisfy

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}-\frac{1}{c_2^2}\frac{\partial^2 w}{\partial t^2}=0.$$

If it is of the form $w(x',y)$ this becomes

$$\frac{\partial^2 w}{\partial x'^2}+\frac{\partial^2 w}{\partial(\beta y)^2}=0,$$

* *Note added in proof.* The limiting velocity is exactly c_r only because we have confined the departure from linear elasticity to a plane.

so that if $$w(x',0)=\int_0^\infty f(s)\sin sx'\,ds, \qquad \ldots\ldots(23)$$

then $$w(x',y)=\int_0^\infty f(s)e^{-\beta sy}\sin sx'\,ds$$

and $$p_{yz}(x',0)=\mu\frac{\partial w}{\partial y}=-\mu\beta\int_0^\infty f(s)s\sin sx'\,ds. \qquad \ldots\ldots(24)$$

If $w(x',0)$ is given by the right hand side of (1) (with x replaced by x') then

$$w(x',y)=\frac{\lambda_0}{2\pi}\tan^{-1}\frac{\beta y}{x'} \qquad \ldots\ldots(25)$$

in agreement with Frank.

If the law

$$p_{yz}=-(\mu/2\pi)\sin 2\pi(w-\bar{w})\lambda_0 \qquad \ldots\ldots(26)$$

is assumed for the interatomic forces across the slip-plane a repetition of the Peierls–Nabarro calculation using (23), (24) and (26) in place of (22) and (21) gives $w=-\bar{w}=(\lambda_0/2\pi)\tan^{-1}(\beta\zeta/x')$ where now $\zeta=\lambda_0/2$ is the width of the stationary dislocation which is thus less than the width of the edge dislocation by the factor $(1-\sigma)\simeq 2/3$. The displacement is obtained by replacing y by $y\pm\zeta$ in (25), the sign of ζ being the same as that of y, i.e.

$$w=(\lambda_0/2\pi)\tan^{-1}\beta y(1+\zeta/|y|)/x'. \qquad \ldots\ldots(27)$$

It corresponds to a distribution of dislocations of the type (25) along the x-axis according to the law (19). (23) can be obtained from (25) by cutting out the material between the planes $y=\pm\zeta$ and sticking together the remainder.

According to (27) the width of a screw dislocation suffers a simple relativistic contraction (with limiting velocity c_2) in complete analogy with the one-dimensional model of Frenkel and Kontorowa (1938).

The calculation leading to (27) is not very significant for a cubic material since there is not a unique slip-plane as there is for the edge dislocation. However (27) may be considered as a crude approximation to the case of a screw dislocation in a hexagonal material. Here there is a unique slip-plane perpendicular to the hexagonal axis. The problem should, of course, be solved using the equations of non-isotropic elastic theory.

ACKNOWLEDGMENTS

The author wishes to thank Dr. F. C. Frank and Mr. F. R. N. Nabarro for helpful discussion and suggestions.

REFERENCES

Burgers, J. M., 1939, *Proc. Kon. Ned. Akad. v. Wet.*, **42**, 293.
Doetsch, G., 1937, *Laplace-Transformation* (Berlin : Springer), p. 102.
Frank, F. C., 1949, *Proc. Phys. Soc.* A, **62**, 131.
Frenkel, J., and Kontorowa, T., 1938, *Phys. Z. Sowjet*, **13**, 1.
Jeffreys, H. and B. S., 1946, *Methods of Mathematical Physics* (Cambridge : University Press), p. 426.
Love, A. E. H., 1944, *Elasticity* (Cambridge : University Press), p. 309.
Nabarro, F. R. N., 1947, *Proc. Phys. Soc.*, **59**, 256.
Peierls, R., 1940, *Proc. Phys. Soc.*, **52**, 34.
Titchmarsh, B. C., 1937, *Fourier Integrals* (Oxford : University Press), Chap. V.

LETTERS TO THE EDITOR

Misfitting Monolayers

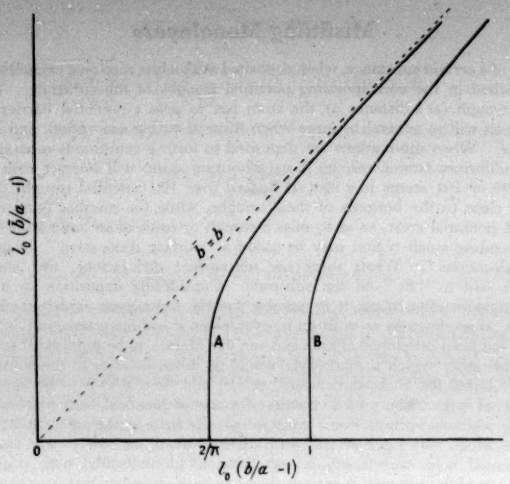
An atom of a certain substance, when deposited at absolute zero on a crystalline substrate, will be adsorbed in the corresponding potential troughs of the substrate. To move to an adjacent trough (at a distance a), the atom has to pass a potential barrier of amount W (say), which will in general be large when there is strong adsorption, and small in the converse case. When more atoms are deposited to form a continuous monolayer, having a natural equilibrium atomic spacing b, neighbouring atoms will interact with each other. When, say, 99 or 101 atoms in a row are spaced over 100 potential troughs, the majority of atoms lie close to the bottoms of their troughs, while the number lying close to and riding over a potential crest, so as to miss a trough or squeeze an extra atom in, is small. The corresponding small region may be called a "surface dislocation", in analogy with "crystal dislocations". When there are no surface dislocations, the corresponding monolayer is said to "fit" on the substrate. Considering deposition as a process of putting down atom after atom, it is possible for the monolayer, under the influence of the substrate, to go down so as to fit on it, even when a and b are unequal.

A theory has been developed (Frank and van der Merwe, to be published) to investigate the conditions under which a monolayer would go down so as to fit the substrate. The theory is developed for a one-dimensional model and then shown to be applicable to a two-dimensional case. The model consists of a row of identical balls (natural spacing b) connected by identical springs (force constant μ), the balls at the same time being acted on by a force which varies periodically with distance along the row. The corresponding periodic potential field, wavelength a, was taken to be sinusoidal with amplitude $\frac{1}{2}W$. The actual shape of the potential curve was shown to be of secondary importance as compared with its maximum variation W, which is the deciding factor.

The mathematical analysis shows that, up to a critical amount of natural misfit defined by $(b/a-1)_{\text{critical}}=2/\pi l_0$, where $l_0=(\mu a^2/2W)^{\frac{1}{2}}$, the state with no dislocations (a monolayer fitting the substrate), is the one of lowest energy. Assuming, for an average case, the interaction between a substrate and deposit atom (expressed in terms of W) to be identical with the interaction of deposit atoms on each other (expressed in terms of μ), calculations with Lennard-Jones forces show that l_0 is about 7, thus making $2/\pi l_0$ about 9%. Beyond this critical value, however, there still remains an activation energy for the formation of dislocations, and this only falls to zero at a larger critical misfit, defined by $(b/a-1)_{\text{critical}}=1/l_0$, which is about 14% for $l_0=7$. Below this critical misfit it is therefore also possible for a monolayer, at low temperatures, to go down so as to fit the substrate. The smaller critical value is presumably important at high temperatures and the larger value at low temperatures. Of course, in practice there will be wide variations about these average values; they will be large when W is large and μ is small, and small in the converse case. Because surface dislocations do not interact until they are close together, there is an abrupt increase in the dislocation density $\bar{b}/a-1$ ($\bar{b}$ is the average spacing of deposit atoms) corresponding to both these states, on passing their critical values, as shown in the graph. Thereafter $\bar{b}/a-1$ is almost equal to $b/a-1$, whence $\bar{b}\simeq b$.

This theory finds an important application in the phenomenon of oriented overgrowth. Crystal orientations are not in general determined by long range forces, but by influences relayed from one monolayer to the next. Hence in order that there shall be a definite orientation in a crystalline overgrowth on a crystalline substrate, there must be formed, as the initial stage, an immobile monolayer of regular atomic pattern, to be called an "embryo". This is exactly the state of the first monolayer if it goes down so as to fit the substrate. Once completed over any flat region of the substrate, an embryo constitutes a suitable substrate, for the formation of another embryo on it. If the atomic pattern of the first monolayer resembles that in a plane of the natural lattice of the deposit, it is possible, by repetition of embryo formation, for a stable strained oriented overgrowth to grow on the substrate. On the other hand the graph shows that, if the critical conditions are exceeded and there are dislocations, their density will be high; the corresponding monolayer, and hence the subsequent growth, will have no significantly preferred location on the

substrate. The strains, permissible in thin layers, cannot, of course, persist in overgrowths of indefinite thickness, since the strain energy involved is much greater. Therefore, at



Graph of $l_0(\bar{b}/a-1)$ against $l_0(b/a-1)$.

A. Lowest energy state; B. State of spontaneous dislocation.

some stage, the abnormal strain energy must be released. At this state the initial orientation may be retained, may be changed, or may be completely lost with the formation of a random orientation. Hence it is likely that the orientations observed in some overgrowths are not the same as those of the corresponding initial embryos. These theoretical ideas are shown (van der Merwe 1949) to be in good general agreement with experimental observations.

We have to thank Professor N. F. Mott for suggesting this problem, and one of us (J. H. van der Merwe) has to thank the South African Council for Scientific and Industrial Research for a grant and special leave, which rendered it possible to perform this research.

H. H. Wills Physical Laboratory,
University of Bristol.
17th February 1949.

J. H. van der Merwe.
F. C. Frank.

Frank, F. C., and van der Merwe, J. H., to be published.
van der Merwe, J. H., 1949, *Trans. Faraday Soc.*, Discussion on Crystal Growth, April 1949 (in press).

The Electron Trap Mechanism of Luminescence in Sulphide and Silicate Phosphors

In a recent publication Garlick and Gibson (1948) report and interpret thermoluminescent measurements on several phosphors. The purpose of this note * is to correlate their experimental methods and conclusions with data independently reported and theoretically analysed by Williams and Eyring (1947). Both papers report and interpret "glow" data on the ZnS–Cu phosphor. Assuming the two samples to be closely similar, a comparison of the experimental and theoretical methods used by the respective investigators can be made.

From their Figure 9 (*b*) showing the variation of thermoluminescence after different times of phosphorescence decay at 291° K., they reasonably conclude from the shift of the single glow peak observed that a distribution of electron trap depths, rather than traps of uniform depth, are active. The uniform heating rate of 2·5 K°/sec., earlier used by Randall and Wilkins (1945) was used. Williams and Eyring reported glow data on the same phosphor but used linear heating rates of 0·0310 K°/sec. and 0·0895 K°/sec. At such slow heating rates, the electron traps of different depths become clearly resolved as reproduced in Figure 1 (Williams and Eyring 1947, Figure 13).

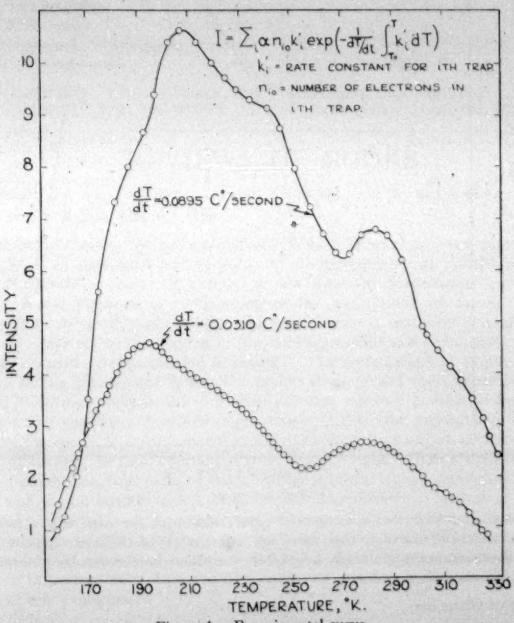


Figure 1. Experimental curve.

By a detailed graphical analysis of the thermoluminescence observed at slow heating rates, Williams and Eyring computed the relative numbers of filled electron traps of different depths and the explicit expressions for the specific rate constants. The analysis can be tested and the experimental results of the two investigators can be correlated by using the relative trap numbers and rate constants obtained from the data of Williams and Eyring to calculate the thermoluminescence data of Garlick and Gibson, substituting, of course,

* Calculations reported in this note were supported by the Office of Naval Research through Contract N7onr–284, T.O.I.

the heating rate and the period of phosphorescence of the latter. At the heating rate of 2·5 K°/sec. and for all phosphorescence periods, all evidence of the fine structure of Figure 1 disappears. This is shown, for example, in Figure 2 for the 20-second phosphorescence period and may be compared with curve *f* of Figure 9 (*b*) of Garlick and Gibson. Differences in peak positions and band width may be attributed to the greater effect at the faster rate heating of competing radiationless processes as evidenced by the dependence of the light sum on heating rates (Cornell 1948), as well as from possible differences in phosphor samples.

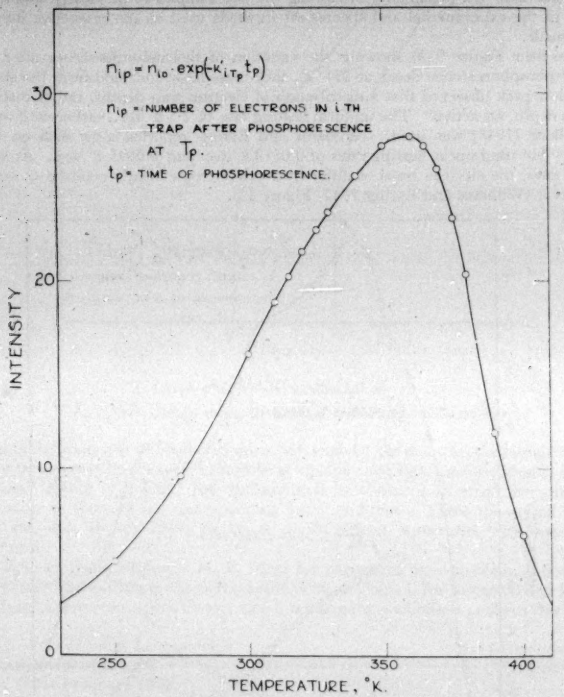


Figure 2. Calculated curve, ~2·5 C°/sec.

It is reassuring that the investigators agree, although for different reasons, on the significant conclusion, namely, that there are often traps of different depths in a single glow peak, that retrapping is usually negligible, and that monomolecular kinetics dominate thermoluminescence phenomena.

Department of Chemistry,
University of North Carolina,
Chapel Hill, North Carolina, U.S.A.
13th January 1949.

J. S. Johnson.
F. E. Williams.

Cornell Symposium, 1948, *Solid Luminescent Materials* (New York and London: Wiley), p. 345.
Garlick, G. F. J., and Gibson, A. F., 1948, *Proc. Phys. Soc.*, **60**, 574.
Randall, J. T., and Wilkins, M. H. F., 1945, *Proc. Roy. Soc.* A, **184**, 347, 366, 390.
Williams, F. E., and Eyring, H., 1947, *J. Chem. Phys.*, **15**, 289.

The significant contribution made by the above letter is contained in the second paragraph. The increase in detailed structure revealed by reducing the warming rate in the

thermoluminescence experiments follows from equation (2) of our paper and the curves of Figure 2 (*a*) since a decrease in the warming rate β produces the same sharpening of the curves as an increase in the constant s.

It is impossible to compare two copper activated zinc sulphide phosphors as suggested in the beginning of the letter. The "glow" curve of such phosphors is markedly dependent on the method of phosphor preparation (cf. glow curve of ZnS–Cu phosphor given by Garlick and Wilkins (1948)). We have "glow" curves for other ZnS–Cu phosphors measured at a warming rate of 2·5°/sec. which show as much detailed structure as those given in Figure 1 above for much smaller warming rates.

The different peak positions of curve (*f*) of our Figure 9 (*b*) and of Figure 2 above may not be correlated since the specimens are different. Even so, we do not think the explanation involving different effects of radiationless processes * for different warming rates can be accepted since, over the relevant temperature range (300–450° K.), there is little change in the luminescence efficiency of such specimens. It is of interest to note that the half width of the curve of Figure 2 is 95° K., although the warming rate is much slower than in our experiments in which the half width of curve (*f*) of Figure 9 (*b*) is 45° K. We suggest that thermoluminescence characteristics need to be studied for very many different specimens of the same type before typical behaviour can be established.

Physics Department, G. F. J. Garlick.
The University,
Edgbaston, Birmingham.
16th February 1949.

Garlick, G. F. J., and Gibson, A. F., 1948, *Nature, Lond.*, **161**, 359.
Garlick, G. F. J., and Wilkins, M. H. F., 1948, *Nature, Lond.*, **161**, 565.

REVIEWS OF BOOKS

Cosmic Rays, by L. Jánossy. First Edition. Pp. xiii + 424. (Oxford: University Press, 1947). 35*s*.

The subject of cosmic rays has a remarkable history, from its inconspicuous beginnings as the study of an unwanted leak in gold foil electroscopes to its present position as a disquietingly generous source of supply of new kinds of fundamental particles. The complexity of the phenomena contributing to the composition of cosmic radiation at various places in the earth's atmosphere, and the difficulty of detection or analysis of some of the effects involved, make progress in this field dependent to a particularly large degree on the improvement of experimental technique. The investigation of atomic and nuclear phenomena in the laboratories led to the design of more and more sensitive apparatus, responding in various ways to the individual particles involved in such processes; and it may be said that each new device or improvement of this type immediately opened up a new field in the study of cosmic radiation and each time disclosed an unsuspected wealth of new phenomena. In fact, cosmic radiation research leads us into a domain of energies which we are only just beginning to reach by means of the largest generators; it has thus been, and still is to a large extent, a most valuable complement to laboratory work on the fundamental properties and the constitution of matter. Its contribution is not restricted to the more spectacular discoveries of new particles and nuclear explosions, but comprises also a considerable amount of quantitative data on the elementary processes concerned, which afford important tests of the validity, under extreme circumstances, of our theoretical treatment of those processes.

One drawback of cosmic-ray study as a source of information on fundamental problems, which applies to all natural phenomena occurring on too large a scale to be under our control, is that we are primarily concerned with the observation of the effects of the impinging radiation without being able to influence their production or essentially to reduce their overwhelming complexity. The most we can do is to try by devious and cumbrous devices to achieve some degree of selection and to disentangle to some extent the maze of atomic and nuclear reactions which build up the radiation as it proceeds through the atmosphere and to great depths underground with continually varying composition. As

* Part of the theory of Eyring and Williams for non-radiative processes in phosphors has been vitiated by experimental facts already given by us (Garlick and Gibson 1948).

a result of this situation, the task facing the author of a monograph on cosmic rays is of disheartening difficulty, even if this author happens to combine, as does Prof. Jánossy, a first hand knowledge of the experimental techniques and a firm mastery of the theoretical methods and results, with a clear and all-embracing view of the vast and steadily growing accumulation of empirical facts and data. From the start he must give up any hope of producing an entirely up-to-date survey; in fact, our author candidly confesses that he has not even been able to harmonize entirely all sections of the book. This last shortcoming, however, is not seriously disturbing; and even though the book fails to give an adequate account of the latest discoveries about mesons, it will undoubtedly render invaluable and lasting services as an authoritative introduction to a subject which is of paramount interest not only to those directly engaged in research in it, but also to all workers in the larger field of nuclear physics.

The book under review covers all the main aspects of the subject, and seems to me on the whole to be well-balanced. It opens with an historical survey, in which the broad outlines of the subject are very aptly presented (the account of the high altitude effects, however, is too sketchy to be of any use). The next chapters are of auxiliary character: they treat the experimental technique and the theoretical background (theory of fast collisions). In this connection one must also mention Appendix I, in which a useful account of the statistical treatment of observations is given. The two following chapters contain a discussion of the phenomena observed at sea-level and underground, and of those connected with the instability of the mesons. Next come a thorough treatment of the cascade theory of the soft component and a comprehensive survey of the geomagnetic effects. The last two chapters are devoted to extensive air showers, and to the origin of the meson component (including an account of penetrating showers). A feature of the book which strikes the outsider on the most casual perusal is the pleasingly moderate size of the list of references: in view of the bewildering and apparently unquenchable flow of publications of the subject, this restraint is an indication of a wholesome critical approach. It is just a little perplexing to find that Prof. Rossi's recent contribution in *Reviews of Modern Physics*, which has the same characteristic, does not present quite the same selection.

It certainly cannot be said of this extensive treatise that it makes easy reading; but I should not like to put the blame for this on the author. He has obviously made strenuous and largely successful efforts to bring a most rebellious subject matter into a well-ordered picture. It is true that the formulation of facts and ideas too often shows less care than one would wish, but these stylistic blemishes, which may easily be remedied in a second edition, do not in general impair the clarity of the account. After all, if the multitude of facts to be kept in mind when following a particular line of argument often strains the memory of the untrained reader, it must be realized that this complexity is an unavoidable feature of the phenomena discussed; and it is nobody's fault if even the best evidence, in spite of the ingenuity of the experimental arrangements, only yields relevant information after being subjected to a subtle and intricate analysis.

In the composition of his book, Prof. Jánossy has paid special attention to the practical needs of research workers: his above-mentioned accounts of experimental techniques, and of the statistical treatment of observations, as well as the numerous diagrams and tables throughout the book will be especially welcome. I would like to present a suggestion in this respect: although the exposition of cascade theory given in the book is very complete, it remains a little difficult, especially for one who does not make daily use of it, to find his way in the maze of formulae and pick out ready answers to specific questions. It would therefore be a great help if a brief treatment of a series of typical cases were added, including the most current applications of the theory with the appropriate references to the relevant formulae.

As to the accuracy and reliability of the information conveyed, a very high standard is maintained almost everywhere; the weakest parts are those concerned with meson theory, which would seem to be in need of a serious overhaul. It was somewhat disturbing to me to find the chapter on the soft component opening with a discussion of the soft component allegedly contained in the primary cosmic radiation. From a didactic point of view, it does not seem advisable to start with such a hazardous illustration of the general theory; altogether, too little emphasis is placed in the book on the controversial character of this particular issue. Dissenting views should certainly have been mentioned and a clear summary of the conflicting arguments given. But we may reasonably hope that no occasion for such a criticism will arise in the next edition!

L. ROSENFELD.

Theoretical Chemistry, by S. GLASSTONE. Pp. viii + 515. (New York: Van Nostrand, 1946.) Third Edition. No price.

Dr. Glasstone's text-book of theoretical chemistry contains chapters which review successively quantum mechanics and the theory of valence, molecular spectra and structure, statistical mechanics and thermodynamics, and intermolecular forces. His object is "to provide an introduction to certain aspects of these subjects that have a bearing on chemical problems", without claiming to give a comprehensive or completely rigorous treatment; he expects any graduate in chemistry to be able to understand the book. It is to some extend a summary of a number of more specialized works, but some references to original papers are also given. On the whole the treatment is formal and there is only the necessary minimum of reference to experimental work. Students will be grateful to Dr. Glasstone for bringing between these two covers so vast and complex a mass of information, competently arranged and making no undue demands on their mathematical abilities. On the other hand the effect is somewhat forbidding, and the pedestrian style does not attract. The book will be useful to those who already have an interest in these matters, but seems unlikely to excite it.

E. F. CALDIN.

Electronics, by F. G. SPREADBURY. Pp. x + 698. (London: Sir Isaac Pitman & Sons, Ltd.) 55*s*.

This is an ambitious book. Mr. Spreadbury has attempted in 698 pages to cover all aspects of the subject. There are twenty chapters, the early ones dealing with the fundamentals of the subject—atomic theory, X rays, electron optics, luminescence, electrical conduction in gases and the like. The author presupposes that the reader has a knowledge of elementary physics and mathematics, including the calculus, and the presentation, so far as it goes, is reasonably clear and accurate.

The later sections of the book deal with applied electronics. The subjects discussed include diodes, triodes, multi-electrode valves, valve amplifiers, oscillators, rectifiers, frequency changers, cathode-ray tubes and applications of these devices in practical circuits. There are chapters dealing with electric discharge lamps, photoelectric tubes, electrical measurements and discussion of various miscellaneous matters including cyclotrons, betatrons and electron microscopes. The book represents a great amount of work in collecting and arranging materials from many sources.

The standard of scholarship varies greatly throughout the book. The chapter on electric discharge lamps is well written and informative, whereas the chapter on electrical measurements is only mediocre. The reviewer has no very clear idea of the type of student Mr. Spreadbury had in mind when he wrote this book. Clearly the latter part of the book is suitable for engineering students taking electronics, although the mathematical approach adopted in the early chapters of the book is, perhaps, not very appropriate to such courses. Nevertheless there is much in the book that engineers will find of value. It is not likely to appeal to physicists. The book is well illustrated and printed and appears to contain few printing errors, although the reviewer noted several errors in the circuit diagrams.

DENIS TAYLOR.

Electromagnetism, by J. C. SLATER and N. H. FRANK. First Edition. Pp. xiii + 240. (New York and London: McGraw-Hill Book Co., 1947.) $3.50.

One's first reaction on seeing a new book on electromagnetism is to wonder whether it is really called for. However, in this instance one's doubts are soon set at rest; this book is certain of a welcome from many students and teachers for its lucidity and for the unusually broad view of the subject that it gives in a volume of fairly small compass. It starts from first principles and covers the whole ground from electrostatics to optics, including electron theory and dispersion, waveguides, cavity resonators, and diffraction, and this in a manner that no one can dismiss as merely superficial. The book is, however, purely theoretical; the very first equation stated is that for the force on a point charge $\mathbf{F} = q(\mathbf{E} + v \times \mathbf{B})$, and the necessity of swallowing this at the first gulp must leave a conscientious student

with a feeling of mental discomfort. However, he subsequently finds his path smoothed as much as possible, and provided he is prepared to take for granted the physical significance of all the quantities, he has every reason to be well satisfied with the comprehensive view that he gets of theory of electromagnetism as it stands today. The authors have written a companion volume on Mechanics, and others are promised; their aim is to bring out the unity of the various branches of theoretical physics; one can only wish the enterprise every success.

It is a sign of the times that the authors, who have previously published a work in the same field in terms of Gaussian units, now use M.K.S. units, and state their conviction that they have advantages even for theoretical work. The movement in this direction, after having been almost imperceptible for nearly 50 years, is now unmistakable, and gives good grounds for hope that the one single system will soon be universally employed and the others allowed to fade away. L. HARTSHORN.

About Cosmic Rays, by JOHN G. WILSON. First Edition. Pp. 144. (London: Sigma Books Ltd., 1948.) 8*s*. 6*d*.

This book is one of the Sigma " Introduction to Science " series, and is meant in the first place for the general reader, though it could also be read with profit by students about to work in the field of cosmic ray physics.

The main criticism is that, through no fault of the author, it is out of date. It is to be hoped that paper can be made available immediately for a revised edition which would include the results obtained, mainly at Bristol, using photographic plates, and an account of the Berkeley experiments on the production of mesons.

This book is logically developed from the Introduction to the apparatus used in cosmic-ray research, to the particles found in cosmic rays in the order of their identification. Included here is a good account of the meson and its connection with the problem of nuclear forces, one of the outstanding topics of physics today.

The last chapter—on Cosmic Rays in Space—could almost equally well have appeared at the beginning. It is so interesting that a shortened version might perhaps have been included in the Introduction.

The drawings and photographs are excellent. E. P. GEORGE.

Tables of Physical and Chemical Constants, by G. W. C. KAYE and T. H. LABY. Pp. vii + 194. (London, New York and Toronto: Longmans, Green & Co., 1948.) 21*s*. net.

This tenth edition of ' Kaye and Laby ' was prepared in Melbourne, presumably on account of the death of Dr. Kaye during the preparation of the ninth edition. It has thus lost some of its associations with the National Physical Laboratory, but it has been fortunate in gaining new assistance in its compilation.

Considerable revision has been undertaken, and in particular the fundamental constants have been corrected in the light of recent determinations and derived constants have been accordingly re-calculated. Similarly, the astronomical constants have been revised, and the Astronomer Royal's determination of the solar parallax is adopted. A useful section on the properties and constants of optical glasses has been added.

A glance at selected tables failed to disclose any misprints, and the general standard of production of the book is very high. H. H. HOPKINS.

Les Théories moleculaires du Pouvoir rotatoire naturel, by JEAN PAUL MATHIEU. Pp. 243. Conferences Rapports sur les Recherches recentes en Physique—Premier Volume. (Paris: Gauthier-Villars, 1946.) 8*s*.

M. Mathieu comments that although his countrymen have made great contributions to the study of optical rotation a collected account of the modern theoretical work on the subject has not previously been available to them. The book is divided as follows: (i) a survey of the early work, (ii) accounts of the general theoretical treatments, (iii) details of the methods of applying the general theories. In general the active substance is assumed to be liquid or gaseous.

It is shown that optical rotation can be explained macroscopically by rewriting the electric displacement equation so as to contain a term in curl $\mathcal{E}$. The coefficient of this term—the "coefficient of rotation"—must be of the nature of a scalar triple product. Microscopically it requires at least four atoms in the molecule to give such a coefficient.

The section on general theory deals with the classical treatment due to Born and de Mallemann; and the wave mechanical method of Condon, rather than the matrix treatment of Rosenfeld. Classically the molecule is assumed to contain a number of coupled vibrating charges. The incident radiation produces the polarization together with a small effect due to the variation of the phase over the molecule. This latter, after averaging over all orientations of the molecule, gives rise to the coefficient of rotation.

In the quantum theory treatment the perturbation of the eigenstates of the molecule is found, and a similar phase effect gives rise to a perturbation term involving the magnetic moment of the molecule. The addition to the displacement vector depends on a product of the electric and magnetic moments, and a similar effect arises from magnetic dipole radiation coupled with an electric dipole perturbation term. The author has neglected the latter effect in the classical treatment, though his argument for doing so is not convincing.

The third section contains a study of the various methods of applying the theory. In order to get a sufficiently great magnetic moment only electronic transitions, or vibrations, are considered. Thus in the quantum theory methods nuclear repulsions, and consequently exchange forces, can be neglected without serious error. Experimental evidence suggests studying the behaviour of radicals, which, though not active by themselves, may become active under the perturbing influence of surrounding radicals.

The molecule of secondary butyl alcohol ($C_4H_{10}O$) is studied in detail. The classical theory method of Boys treats the molecule as being composed of four isotropic dipoles, and considers their mutual interactions. The quantum theory method of Kirkwood, which computes the matrix elements under a dipole–dipole interaction between certain radicals, gives a result which is comparable with that of Boys. Kuhn regards the active part of the molecule as composed of two anisotropic dipoles. He is led to deduce a configuration of the molecule which is the mirror image of that given by Boys and Kirkwood.

Finally, there is the mono-electronic method of Condon, Eyring and their co-workers. They consider a single electron of the OH radical and enumerate its possible transitions under the influence of the radiation, allowing for the effect of the surrounding atoms. Their result shows that the rotatory power varies greatly, and may even change sign, when small changes are made in the configuration of the molecule, or when different types of perturbing forces are used.

It seems that, in spite of the beauty of the general theories, their application is in a very primitive stage of development, due chiefly to the serious mathematical difficulties. M. Mathieu's book should, however, be of considerable value to workers in optical activity; and it should also be of interest to a wider public, partly because the methods of attacking the problem have much wider fields of usefulness, and partly because the development of the theory is an interesting commentary on the growth of modern atomic theories.

J. HAMILTON.

Bulletin Analytique. Vol. VIII, No. 11–12, Parts I and II. (Paris: Hermann et Cie, 1947.) Part I, pp. 2577–2964; Part II, pp. 2025–2345.

What *Science Abstracts* attempts to do for Physics, and *Chemical Abstracts* for Chemistry, the *Bulletin Analytique* undertakes for the whole range of Science—Physical and Biological, Pure and Applied. It takes nothing less than the whole of scientific literature for its domain. This colossal venture is in the hands of a sub-section of the *Centre National de la Recherche scientifique* known as the *Centre de Documentation*, a special branch of which carries out the actual editorial duties. The cover also bears the imprint of the Ministère de l'Éducation Nationale.

Of the two volumes of the double number which have come to hand, the first (Ire Partie) deals with the physical sciences and their applications, the second with the biological sciences and their ancillary techniques. One imagines, from the pagination, that each "Partie" is intended to form its own separate volume at the end of the year. The two parts together comprise rather more than 700 pages, and include more than 8,000 abstracts

in all. It is a sobering thought that, on the evidence of this journal, scientific papers are being turned out at the rate of one every ten minutes, night and day, day in and day out!

In style and length the individual abstracts are similar to those with which we are familiar in *Science Abstracts*: adequate to indicate the scope of the publication, but not to absolve the reader from the necessity of consulting it. In this connection, however, the Centre de Documentation offers to supply readers with a microfilm reproduction of any of the papers abstracted, a scheme which may be commended to the attention of the learned Societies responsible for the publication of our own *Science Abstracts*.

One gathers that the *Bulletin Analytique* culls its extracts from a rather wider field than our own abstracting journals, and that, in addition to the standard scientific journals, technical, semi-popular and even trade journals are scanned for matter worthy of note. One learns, for example, of the part played by amateur radio fans during the Texas disasters; that geologists have made an excursion to Reading; that Plastics has been looking at Brewing, and that, in view of the recent drought, Valparaiso is to accelerate its plans for a new water supply.

A very valuable addition to the normal abstracting service is the list of new books of scientific and semi-scientific interest which appears at the end of each Part of the *Bulletin*. An author index is also included with each part. Our French colleagues have no cause to complain of the efficiency of their abstracting service. J. A. CROWTHER.

Heat, by A. G. WORTHING and D. HALLIDAY. Pp. xii+522. New York: John Wiley; London: Chapman & Hall, 1948.) $6.00.

In recent years there has been a big output of books dealing with atomic physics, electronics and cognate subjects, but comparatively few on the older branches of physics such as Heat. The volume under review helps to restore the balance and will probably give the same pleasure to many readers as the appearance of Preston's *Heat* did to their fathers and grandfathers about half a century ago.

One of the authors, Professor Worthing, has an international reputation for his contributions in the high temperature field, and in the present comprehensive treatise he and his co-author give an admirable survey of the present state of our knowledge as regards temperature measurement, expansion, calorimetry, specific heat, thermal conduction, properties of gases, thermodynamics, change of phase, heat engines, refrigerators, convection and radiation.

It is a textbook intended for graduate courses in American universities, but it is also certain of a hearty welcome on this side of the Atlantic.

The authors have searched the literature for new experimental methods and have thereby helped the student, and also the research worker, by providing a first reference to modern methods.

It might be remarked that the authors have taken great care to be precise and consistent in their definitions and in the use of terms and symbols. It will be interesting to see what reception is given to their attempt to eliminate much of the uncertainty in students' minds as to the distinction between mass and weight. They propose the verb "to mass" to apply to the action when the analytical balance is used, and the verb "to weight" is reserved as appropriate for the action involving the spring balance.

A few words will cause mild surprise, for example the statement "there has been much *revamping* of old concepts since the introduction of Planck's quantum concept in 1900", and later, in connection with the porous plug experiment, to be told expansion does work on the gas at the lower pressure in *shoving* it out of the way. The italics are the reviewer's.

That the authors are up to date in their subject is evident throughout the book. One instance may be quoted: in a footnote to a table giving melting points and freezing points, they indicate the change of melting points of certain metals which will result if a new value is decided upon for the second radiation constant c_2. The Consultative Committee on Thermometry of the International Bureau of Weights and Measures which met in Paris in 1948 has decided on a new value for c_2, particulars of which will be published shortly.

In a work of this magnitude it is inevitable that a few errors should creep in, and they are minor blemishes, noted here to enable the reader to correct his copy of the book and perhaps help the authors in the preparation of a new edition.

The optical lever method described for measuring coefficients of expansion was devised by Roberts whilst at the N.P.L., not the Cavendish Laboratory.

In the diagram on page 306 the heating coil is shown extending on to the area of the bulb in contact with vapour which might result in superheating.

The apparatus shown on page 174 for the determination of the thermal conductivity of a metal in rod form gives inaccurate results when applied to good conductors; it may, however, be used for carbon rods. Consequently the curve on page 176 giving the variation of thermal conductivity of nickel with temperature is incorrect, as in the case of nickel the conductivity increases with temperature from the change-point.

Two additions might be suggested in future editions. On page 41 one would like to see the simplest form of thermoelectric potentiometer described as an introduction to the more complicated circuits.

On page 72, in connection with the linear expansion apparatus, it would be helpful to mention the dial gauge which is so much used nowadays in this form of apparatus.

The volume can be recommended to students as an excellent treatise on Heat, and the authors are to be congratulated on performing a very useful service to both British as well as American students of the subject.

EZER GRIFFITHS.

Colloid Chemistry, by R. J. HARTMAN. Second Edition. Pp. xxxii+572. (London: Sir Isaac Pitman & Sons, 1948). 55*s*.

The chapters in this volume are conveniently grouped into sections. After a brief introduction, six chapters are devoted to the subject of surface chemistry, then follow ten chapters on lyophobic and four chapters on lyophilic colloidal systems. The last six chapters describe the biocolloids and some colloidal aspects of organisms.

In the first section where the author discusses the scope of colloid chemistry and the subdivision of mass, attention is drawn to the solubility relationships of small crystals. It is a pity that the difficulties which arise in this somewhat naive picture of crystal equilibrium are not even noted. Quite recently, problems of crystal growth have aroused great interest and whilst reference to the Gibbs vapour pressure surface tension equation is justified, attention should likewise be drawn at the same time to Gibbs' discussion on crystal equilibrium.

In the chapters dealing with adsorption, whilst the early work of Freundlich and Langmuir are given, as well as a note on the B.M.T. and the Harkins–Jura method of treatment, the more recent developments, such as the contrast between immobile (Langmuir) and mobile monolayers (Volmer) or monolayers with inter-attractive or repulsive forces are nowhere referred to. A very readable account of Langmuir's work is given; some of the more recent work of English workers in the field of friction and lubrication, e.g. of Bowden, might well have been included. This section concludes with a brief and general account of contact catalyses.

The author adopts the convenient classification of W. Ostwald and of von Buzagh in classifying colloidal systems according to their phase and dimensional characteristics, and in this section gives a very readable account of the various methods, both physical and chemical, of preparing colloidal dispersions.

In discussing the physical properties of colloidal suspensions the work of Odén on settling, of Perrin on Brownian movement and of Svedberg on the ultracentrifuge, are mentioned, as well as a useful section on Debye's extension of the work of Tyndall and Rayleigh. Only the early work on the electrokinetic properties of suspensions is given. This section concludes with a general descriptive treatment of dialysis mainly technical in character, of aerosols, emulsions and foams.

The section on lyophilic colloids and the gels is introduced by a consideration of methods for determining the surface tension of solutions; dynamic methods are not included.

The lack of preciseness in thought, which some years ago was characteristic of the subject of colloids as a whole, is nowhere so well exemplified as in our present ideas regarding

the structure of gels. Chapters 19, 20 and 21 contain a large volume of descriptive matter brought within small compass and it is evident that much more work must be carried out on these systems to clarify our ideas.

The author has chosen an excellent structure for a book on colloid chemistry. It is well printed and contains a great deal of information of value to those who are particularly interested in the technical aspects of colloid chemistry. ERIC K. RIDEAL.

The Optical Principles of the Diffraction of X Rays, by R. W. JAMES. Pp. xv + 623. (London: G. Bell & Sons, Ltd., 1948.) 80*s*. net.

When the study of the diffraction of x rays by crystals was initiated in 1912 by the Laue – Friedrich – Knipping experiment, it was the physicists who were principally interested; experimental and mathematical physicists—and for a very good reason. X-ray diffraction is a branch of optics; and none but physicists were properly trained to appreciate and overcome all the initial difficulties. The first few simple structures that were determined showed clearly that both crystallographers and chemists would benefit greatly by the knowledge acquired through these experiments. But it was really not until some fifteen years or more had passed that the experimental difficulties had become sufficiently well understood and techniques standardized for the real research interest to move towards the field of structural chemistry.

It is perhaps surprising therefore that the first textbook in English dealing comprehensively with the optical principles of the diffraction of x rays should not appear until more than a third of a century after that auspicious beginning. For a great deal of the experimental spadework was carried out in England, especially in the school of Physics at Manchester, under the leadership of Professor W. L. Bragg. No doubt one reason for the delay has been that those who were capable of writing such a book, and they are not many, were more interested in continuing with their research work. Yet there comes a point when, for the sake of succeeding research students who have not had that early important training (some, indeed, of whom have had far too little training in optics) it is very desirable that a rigorous and yet readable exposition of fundamental principles should be made available in book form.

Professor R. W. James was one of the earliest and most active workers in the field of x-ray optics and, in spite of the difficulties of war-time isolation in South Africa and of heavy teaching and administrative duties, he has maintained an unbroken personal interest in experimental crystallography. No one could be better fitted to write such a textbook, and no one could have written it better.

This volume is not concerned with actual experimental techniques as such, nor with the results of structural research; but the general optical principles with which it deals are well illustrated by reference to experiment in a way that only an experimental physicist could have achieved. At the same time the sound mathematical presentation will provide excellent material for study by students who want to get down to fundamentals. Experienced research workers are sure to refer to it constantly.

The main subjects considered are the geometrical theory of diffraction by space-lattices, both simple and subject to various perturbations; the intensity of reflection of x rays by perfect and by mosaic crystals; Ewald's dynamical theory; the atomic scattering factor; anomalous scattering and dispersion of x rays; the influence of temperature on diffraction; experimental problems of primary and secondary extinction; the use of Fourier series (including Patterson's series) in crystal analysis, and the Fourier projection considered as an optical image; Laue's development of the dynamical theory and its application to Kossel lines; the scattering of x rays by gases, liquids, amorphous solids; diffraction by small crystals, effect of crystal size and of certain kinds of faults and imperfections.

Since the vector notation is used throughout the book, a summary of vector formulae is given in an appendix. Other appendices give a brief account of the geometry of the reciprocal lattice, tables for estimating the corrections for dispersion which must be applied to the atomic scattering factor, and one example of the derivation of a Fourier integral.

Perhaps in a book of this extent it is unreasonable to ask for more, but it would be useful to have, in a second edition, an account of the phenomenon known first as " unweganregung ",

sometimes called the "Renninger effect", which is of particular importance in connection with the intensity of the "forbidden" 222 reflection from diamond. (Incidentally, Renninger's name is misspelt in all the places in which it occurs in the book.) It would be helpful also to have some consideration given to the case where multiple reflection occurs within a mosaic crystal in which the individual blocks, although too small to show primary extinction, are in fact parallel; such cases do occur quite frequently in practice and lead to a relative diminution of the secondary extinction in the case of the stronger spectra. The section dealing with the optics of diffraction by lattices in which the arrangement of scattering material is only statistically periodic may need some enlargement in a later edition also, especially for the case of single crystals large enough to give oscillation or Weissenberg photographs.

It only remains to be added that the book is excellently produced and that there is a detailed Table of Contents as well as a subject and an author index. K. LONSDALE.

Kinematic Relativity, by E. A. MILNE. Pp. vii+238. (Oxford: University Press, 1948). 25*s*.

Professor Milne's new book is called a sequel to *Relativity, Gravitation, and World Structure*. It is, however, intended to present the case *ab initio*, and the reader can follow it without necessarily being familiar with the earlier book or the intermediate papers.

The fundamental idea is the comparison of distant clocks by means of light signals, observers on different particles being able to observe each other's clocks. If a signal leaves A and is reflected from B back to A, A can set up a correspondence between the two times on A's clock and the time shown on B's clock at the moment of reflection. A and B can, it is shown, graduate their clocks so that the relations are symmetrical between A and B. The time of travel is interpreted as a distance in terms of a conventional velocity of light. A set of observers is then considered such that if a ray from A to B, say, would meet C on the way, then the return ray would also meet C. This evidently postulates a relation between the motions of A, B and C. Collinear sets in different directions are compared, and it is found that the results correspond to a set of particles expanding symmetrically from a common origin, and that coordinates can be introduced for each particle, those used by different observers being connected by Lorentz transformations. Distance is not supposed measured by a separate scale but defined in terms of travel times. It is then found that a further regraduation of clocks is possible such that the time t of each observer is replaced by τ, where

$$\tau = t_0 \log (t/t_0) + t_0.$$

With a corresponding rescaling of distance, the interval between neighbouring events ds can be expressed in terms of coordinates with the same values for all observers on fundamental particles. In this form it represents a hyperbolic space.

Milne then considers the distribution in a substratum composed of fundamental particles, and assumes that every observer will find the same law of density about himself. This leads to a density law for the particles and to an infinite mass for the substratum as a whole. He proceeds to a discussion of a free particle, that is, one whose velocity is not the same as that of a fundamental particle at the same place. Dimensional considerations lead to a considerable restriction on the possible forms of the acceleration. By an indirect argument that Milne himself seems to find obscure he finds a definite answer in Lorentz-invariant form, which becomes in τ measure an inverse square law of gravitation. An energy function is found, which is conserved, has the same value for all observers, and is not simply the t component of a 4-vector. The main result is that τ measure corresponds to Newtonian time, but is less fundamental than t measure because it contains the arbitrary constant t_0.

Applications are made to spiral nebulae, electromagnetism, and classical atomic theory. Modifications of Maxwell's theory lead to a force law between two point charges that involves the classical radius of the electron explicitly, and to a change in the magnetic interaction, which does not depend on interpretation by spin. An adaptation to Bohr's theory gives two distinct sets of circular orbits, one of which suggests the neutron.

The theory invites comparison with that of Eddington, though the results differ greatly. The important question with both, I think, is how much good physics is buried under the bad epistemology. Milne is certainly the more intelligible, but then he does not tackle modern quantum theory.

In the first place, what are the fundamental particles? Milne emphasizes (p. 10) that the process is akin to the construction of an abstract geometry, and makes no appeal to experience; it is sufficient that the structure is self-consistent and free from contradiction. (The distinction is too delicate for me.) Then the fundamental particles can be imaginary points carrying imaginary clocks, and we are at liberty to make them move in any way we like. All that Milne shows about them is that it is self-consistent to suppose that they all move uniformly in straight lines from a single origin. But he later identifies them with the nuclei of extra-galactic nebulae (and presumably the clocks with Cepheid variables), and then the rule that A, C, B, once collinear, remain collinear, becomes an empirical postulate. With some forms of the law of gravitation it would clearly be false. Suppose that gravitation is Newtonian but has no effect on light; that the mass is mainly concentrated near the centre; that there are outlying particles at A, B; and that the normal to AB from the centre meets AB at C between A and B. Then the acceleration of a particle at C would differ from that of any point on AB. But such a scheme is certainly self-consistent. Again, there is no objection to a set of ideal points being distributed so that the distribution appears the same from every point; but ideal points have no mass, and the argument for the infinite mass of the universe breaks down for them. Milne is sure that the mass of the real universe is infinite, this being one point where his system differs seriously from general relativity, but this is an empirical hypothesis and not an abstract rule.

The argument for the inverse square law is dimensional. The problem is to construct out of position and velocity a 4-vector that will have the dimensions of another 4-vector that depends on the acceleration. It is assumed that the only invariants concerned are those already found in the theory. If there are others, the argument breaks down. For instance, if there is anywhere a standard metre bar, a characteristic time could be determined, and could appear in the dimensional formula, which therefore begs the question. The consequences of this type of dimensional argument were fully exposed by N. R. Campbell many years ago, but it continues to be taught and to work havoc in physics. It is conceivable, as for the square law of resistance in hydrodynamics, that the argument may give approximately correct results over a certain range of values of the variables, but it should never be taken as more than a suggestion.

Mach's principle, several times appealed to by Milne, has never, so far as I know, been stated in such a form as to be of any use. In practice, like the dimensional argument, it is made to say that nothing matters except what the investigator is taking into account, and hence that any guess is certain on "philosophical" grounds; hence the number of mutually inconsistent theories of the expanding universe now current.

The most disappointing feature of the book is that, though an exact form of the law of gravitation on Milne's theory is given, its consequences are still not worked out. If it provides explanations of the motion of the perihelion of Mercury, the eclipse displacement, and the spectral shift in the Sun and the companion of Sirius, then it is consistent with Einstein's on the solar system scale; if not, it is physically wrong. I have often found in conversation with physicists that they think that the verification has been carried out because it is so obvious that it ought to be.

On the other hand it is satisfactory that Milne has found equations of motion consistent with the special theory of relativity, which may be useful in problems where gravitation can be neglected, and his modifications of Maxwell's equations and the law of magnetic interaction certainly deserve further examination.

HAROLD JEFFREYS.

Fourier Technique in X-Ray Organic Structure Analysis, by A. D. BOOTH. Pp. viii + 106. (Cambridge: University Press, 1948.) 12*s*. 6*d*.

Structure determination is an important, rapidly advancing, complex art, and Dr. Booth sets out to guide the uninitiated along the tortuous paths that lead from a table of experimental structure amplitudes to a complete knowledge of the atomic arrangement within the unit cell. The attempt is timely, as there is no modern critical account of the many methods that have been devised, developed, modified or abandoned in the last twenty years. The considerable delay between the completion of a manuscript and the appearance of the printed book has, however, left its trace, so that direct methods of phase

determination and the method of steepest descents receive only a mention in the preface. For those with a fair acquaintance with the subject Dr. Booth is a good guide. Nearly every development in Fourier technique is mentioned, and the comments on their relative usefulness are just. Particularly welcome is a connected account of Dr. Booth's own contributions to the subject.

The main chapters of the book are: Methods of obtaining approximate structures, The refinement of atomic coordinates, Methods of computation, and Mechanical computation. There is a certain lack of balance in the treatment; over seven pages are devoted to the theory of the "fly's eye", and less than four to the use of the Beevers and Lipson strips. There is very little about the practice of the fly's eye, and only an exceptional research student would feel no need to seek further guidance on the use of the strips. In execution, moreover, the book leaves much to be desired. Perhaps the least satisfactory part is the first chapter, The interaction of X rays with matter. Dr. Booth assumes from the beginning that the reader has a basic knowledge of classical crystallography and X-ray diffraction. Even so, this chapter suffers from a terseness that will make it difficult for those not already well acquainted with the subject, and also from a certain sloppiness in notation. In equation (1.2) a single prime is simply a distinguishing mark, but a double prime denotes the second derivative with respect to the time. In equation (1.12) the letter A is used with two meanings, one defined only by implication, and later N is used with two meanings in successive equations. Such examples could be multiplied, each trivial, but the cumulative effect is irritating. Orthogonal axes are tacitly assumed, and the physical meaning of the mathematical device used to evaluate the integrated intensity of reflection is left obscure.

The book can be thoroughly recommended to the experienced crystallographer. The research student will have to look elsewhere for physical background and practical details of certain techniques, and would be well advised to leave the perusal of this book to a late stage in his training.

A. J. C. WILSON.

Wave Mechanics and its Applications, by N. F. MOTT and I. N. SNEDDON. Pp. xii + 393. (Oxford: University Press, 1948.) 30*s*.

This book will be a welcome addition to the library of the student of physics or chemistry, covering as it does many applications of wave mechanics of great importance which are normally only found in specialist monographs or original papers and not in the textbooks on wave mechanics. The authors' aim has been to write a book for those who want to *use* quantum mechanics. They therefore deal only briefly with the foundations of the subject and devote most of the book to techniques of application and results. The book is nevertheless self-contained and develops logically from its first chapters. The treatment is everywhere concise, and the reader is in no danger of being bogged down by detail. There are copious references to the original papers for anyone wishing to study further. The emphasis, as the title implies, is on wave-mechanical methods, but matrix methods are also used where appropriate, and a chapter at the end of the book is devoted to the historical development of matrix mechanics, thus rounding off the logical form of the exposition.

The book covers such applications as chemical binding, interatomic and intermolecular forces, the theory of solids, collision problems, relativistic wave mechanics and the interaction of radiation with matter. It also gives a brief account of modern developments in the still open fields of the theory of elementary particles and quantum electrodynamics: this is of necessity so superficial that one may doubt whether it was worth doing. Among the useful and unusual features is a chapter on perturbation methods, in which not only the usual straightforward developments in power series are given, but also more refined methods which have been found useful in applications and which are less generally known. The section dealing with the self-consistent field methods of Hartree and Fock, and the Thomas-Fermi model, should also prove welcome.

In the attempt to be concise some misleading statements have crept in. For instance, in the section on magnetism the term in the wave equation which is quadratic in the magnetic field is neglected. This is the term which gives rise to diamagnetism. Paramagnetism

is fairly fully discussed but diamagnetism is ignored ; later, however, evidence on agreement between calculations based on Hartree and Fock wave functions and observed diamagnetic susceptibilities is quoted, which might well puzzle a reader not familiar with this aspect of the subject. The reviewer also found the section on the relativistic treatment of the motion of an electron in a magnetic field confusing, giving as it does a particular solution of the wave equation whose physical context is not discussed. These, however, are minor blemishes. The book is surely one of the most useful published recently. M. H. L. P.

Micromeritics, by J. M. DALLAVALLE. Second Edition. Pp. xxviii + 555. (New York: Pitman Publishing Corporation, 1948). 42*s*. 6*d*.

The author suggests a new word, *Micromeritics*, to mean the science and technology of fine particles, and gives his suggestion weight by using the new word as the title of a very fine book. No one could quarrel with the Greek employed in this synthesis but, on the other hand, the sub-title, "The Technology of Fine Particles", is plain English and describes the book exactly.

Expensive scientific books are not usually set out to catch the eye. This one, like other books from the same publishers, is different. The bright red, white and black dust cover and the cream boards will inevitably stimulate curiosity in a laboratory or scientific library. The agreeable appearance is backed up by excellent paper, printing and diagrams. Perhaps partly in fun, but none the less with sound business judgment, the printers and publishers have subjected the scientists to the powers of modern advertising, and they have done this with good taste. The book is as smart as a new chromium desk lamp.

As for the contents of the book, they resemble an encyclopaedia. The twenty-three chapters are distinct and each covers a wide field. Broadly speaking, the arrangement is as follows. First comes the dynamics of a single particle in a viscous medium, including applications such as the centrifuge. The next seven chapters deal with the shapes of particles distribution functions, the methods of particle size measurement, the theory of sieving and grading, packing, voids and the behaviour of aggregates under pressure, diffusion of particles and the electrical and sonic properties of particles in a carrying fluid. The following six chapters describe the heat conductivity of packings, the relationships between surface energy, heats of wetting and the like, special aspects of the physical chemistry and chemistry of small particles such as crystal growth and oxidation, the flow of fluids through packings, particle-moisture relationships and capillarity. Chapter 16 describes methods of measuring particle surface. Chapter 17 is about muds and slurries. Chapter 18 summarizes work on the transport of silt in rivers and of muds and slurries through pipes and nozzles. The following three chapters are about dust storms, dust in the atmosphere and industrial dust, and the methods of separating dust from air. Chapter 22 is a short account of fine grinding. The last chapter discusses the application of statistics to sampling as well as giving a critical commentary on the experimental procedure. Finally, there is a bibliography containing a list of over 400 papers, seven appendices, an author index and a subject index.

The author has spared no pains to make the book comprehensive and quickly usable. The list of contents gives chapter headings and paragraph headings. There is a four-page list of the principal symbols and the dimensions of the quantity symbolized. The indexes are excellent, and the bibliography quotes all titles in full in their original language (sometimes with a translation) and the exact reference details. Each chapter is concluded with about half a dozen problems. Some are merely numerical substitutions in formulae given in the text; others are more general and do illustrate and amplify the text. The reader is also helped, where necessary, by the inclusion of sections summarizing fundamental ideas lying at the fringe of the main subject. For example, ten pages are given to the theory of diffusion by turbulence; several pages are given to the various corrections to Stokes' law; and the theory of the coagulation of smokes receives a section on its own. Enough is said to make clear to the reader that if he has a special interest in these matters, a study of the original literature is unavoidable.

The book contains a very large number of equations, and about twenty per chapter are considered sufficiently important to justify a number. Sometimes the equations are derived from commonly known laws, but frequently they are quoted from original papers as representing the best fit to experimental results. The more academic reader might feel a little

uneasy at finding on the one hand equations with a sound fundamental basis, such as the Rayleigh scattering formula, and on the other hand empirical equations, such as Gilbert's, for the amount of silt moved by a stream. The scope of the book, however, unavoidably reaches into regions of science where progress depends more on experiment than on abstract contemplation. The author often does what he can to indicate the reliability and range of the empirical formulae, and sometimes makes a short but valuable criticism. The monumental effort that he has made in reading and summarizing a voluminous and scattered literature deserves our thanks and praise.

The book is highly recommended for all scientific libraries used by chemists, physicists, mathematicians and civil engineers. Physiologists also will find sections of the book informative. Industrial research laboratories encountering problems of dust hazards will probably already have the works and technical side covered comprehensively, and the value of the book to them will lie in its description of scientific fundamentals. W. G. P.

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ABSTRACTS FOR SECTION B

Vibrations of Free Rectangular Plates, by Mary D. Waller.

ABSTRACT. Records are given of the normal vibrating modes and frequencies of free rectangular plates between the limiting shapes of the bar and the square. The nodal systems, which in general consist of straight lines parallel to the sides, are, from considerations of symmetry, divided into four classes. Combined modes, for which the nodal patterns are less simple, are not uncommon. The constituent modes belong to the same class, but their uncombined periods may be appreciably different. The combination of modes belonging to different classes is extremely rare, the uncombined periods differing very little in frequency. As the mirror symmetry of the nodal design is lost in such combinations, it may be questioned whether they are ideally possible even for modes of exactly equal period.

On the Pulse Method of Measuring Ultrasonic Absorption in Liquids, by J. M. M. Pinkerton.

ABSTRACT. This paper deals with the experimental problems involved in accurate measurement of the absorption of ultrasonic waves in liquids. Reasons are given for preferring a method employing pulses of ultrasonic energy. The errors likely to be introduced by diffraction are discussed and it is shown that reliable measurements may be

made in both the Fresnel and Fraunhofer regions. An account is given of a convenient method of correcting for divergence of the beam in the Fraunhofer region. The choice of the optimum conditions for accuracy is discussed and illustrated by practical examples. A description is given of the essential features of an apparatus working on six frequencies between 7·5 and 67·5 Mc/s. using the pulse technique.

The Confinement of Slow Charged Particles to a Toroidal Tube by J. W. GARDNER.

ABSTRACT. In the magnetic field of an infinite straight filament current a slow charged particle is confined near the current but drifts parallel to it. This suggests the field of a circular loop of current as a means of confining a particle within a toroidal tube, and it is found that a particle is so confined in such a field if its velocity is sufficiently small in relation to the field. The results obtained are used to examine briefly the possibility of a magnetic self-constricting effect in electric sparks and arcs.

Observations made on the Ionosphere during Operations in Spitsbergen in 1942–43, by A. B. WHATMAN.

ABSTRACT. Observations on the ionosphere were made in Spitsbergen (latitude 78° N., longitude 15° E.) in 1942–43 on behalf of the Admiralty. Obserxations in such a high latitude have seldom been made. The salient features of the equipment and site are described, the equipment proving excellent except for the magnetograph.

Each region of the ionosphere is then considered in turn; the same main regions are found in Spitsbergen as elsewhere, but there are many abnormalities, of which the "Polar Spur" is perhaps the most interesting. Ionization often changes with great rapidity. The effects of magnetic storms are also described.

The Scotopic Visibility Function, by B. H. CRAWFORD.

ABSTRACT. The scotopic (dark-adaptation) visibility of radiation through the spectrum has been determined for fifty observers under 30 years of age. A modified photometric matching method was used at a very low brightness (3×10^{-6} candles/ft^2 or $3{\cdot}2\times10^{-9}$ stilbs). A subsidiary investigation showed that the ultimate scotopic curve was approximated closely. Another demonstrated the effect of age, showing a progressive decrease in sensitivity at the blue end of the spectrum with increasing age, the effect first becoming noticeable at about 30 years of age. A detailed comparison of the present results with those of previous workers is made and reasons for discrepancies are discussed.